Computer Graphics

Mesh Processing

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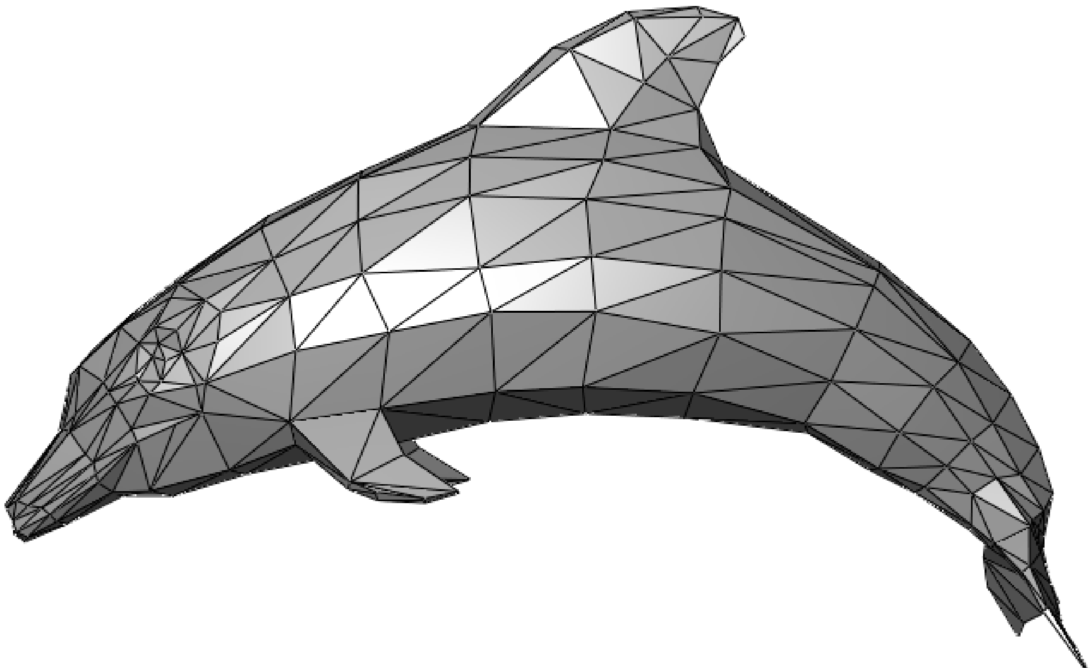
What is Polygon Mesh?

- A polygon mesh is a collection of vertices, edges, and faces that defines the shape of a polyhedral object in 3D computer graphics and solid modeling.

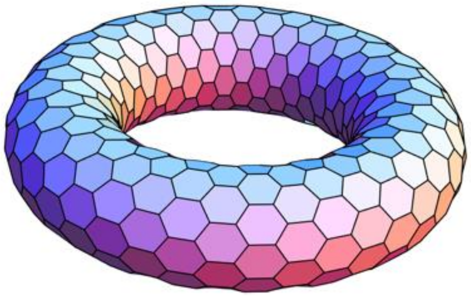


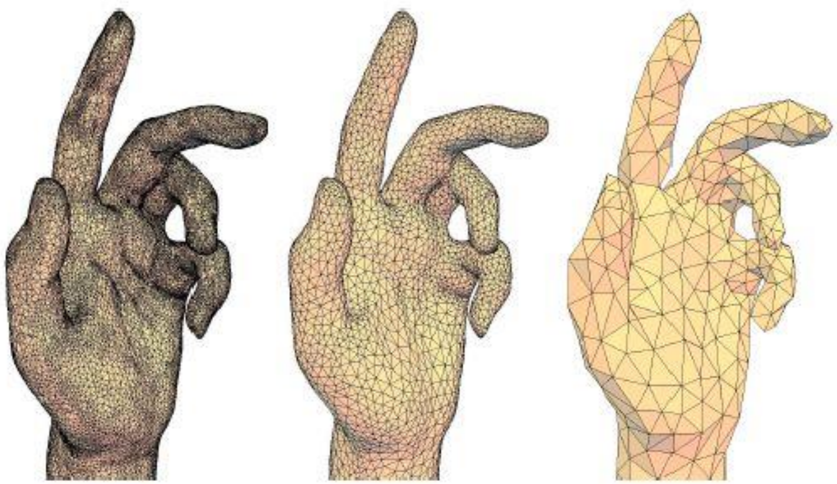
Example – Polyhedral widgeon

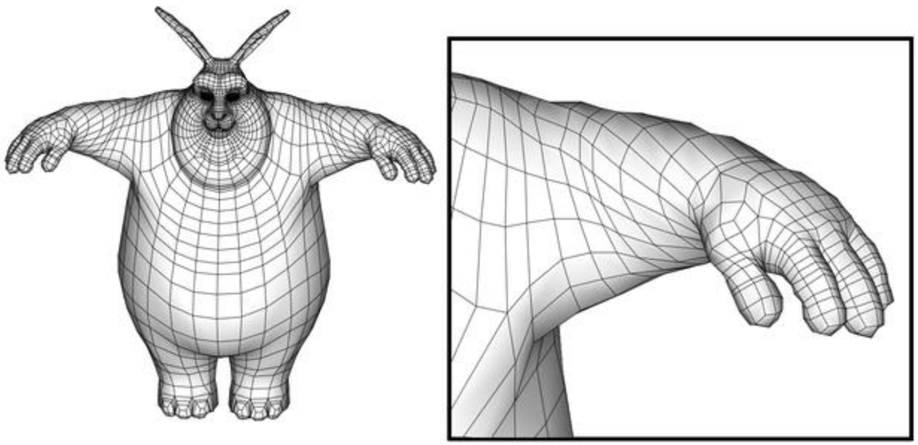
6656 faces (面)，3474 vertices (顶点)

Categories of Polyhedron (多面体)

- Polyhedron are essentially linear approximation
  - Triangular meshes (三角网格)
  - Quadrilateral meshes (四边形网格)
  - Polygonal meshes (多边形网格)



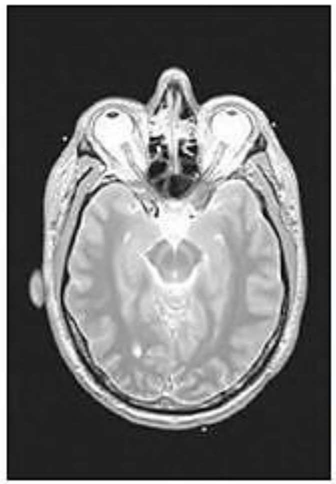




Volumetric Scanning

- Build voxel structure by scanning slices

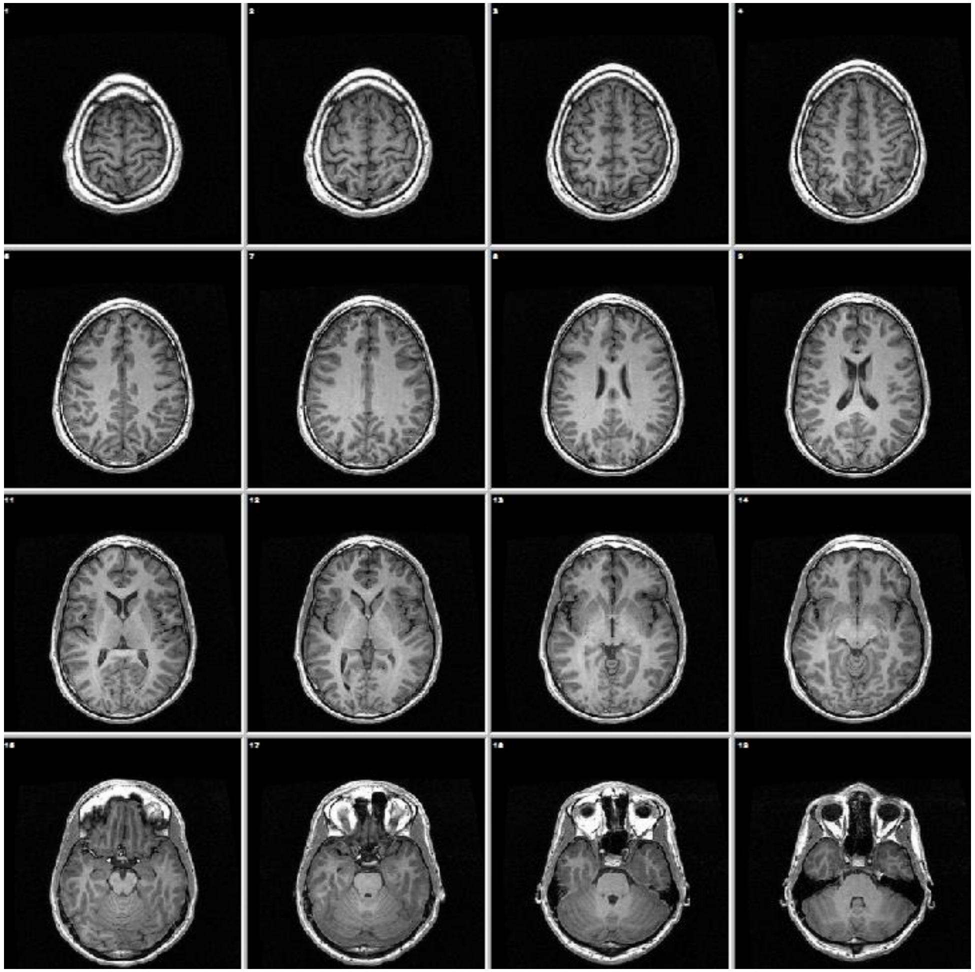
CT

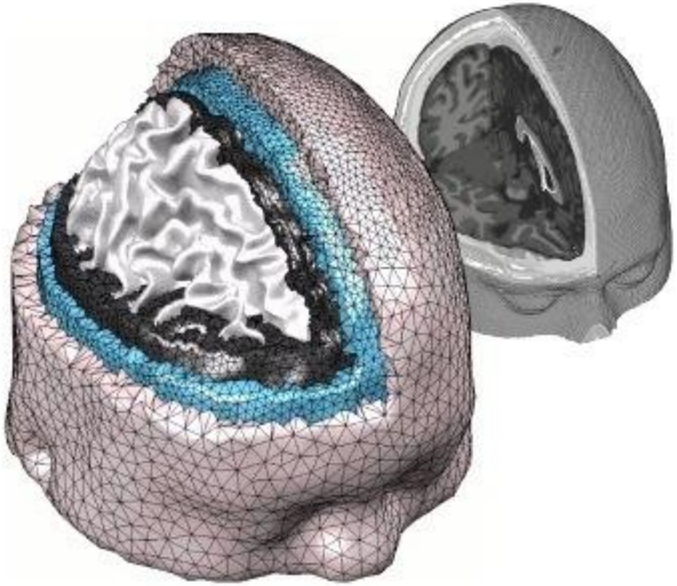


MRI

Volumetric Scanning

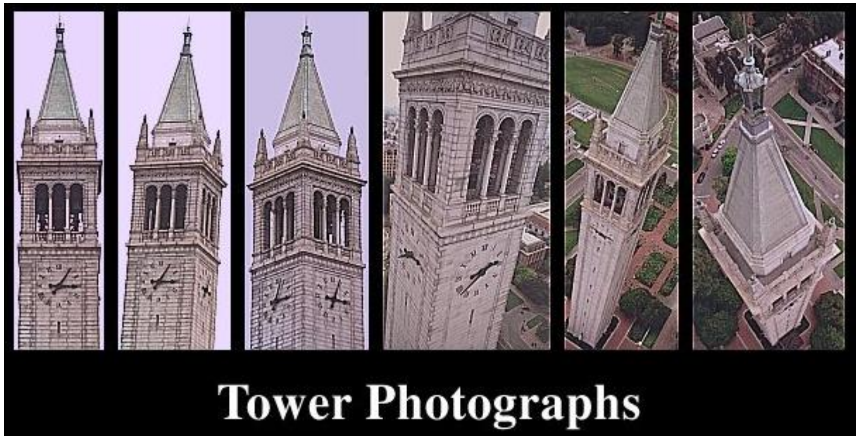
- Build voxel structure by scanning slices

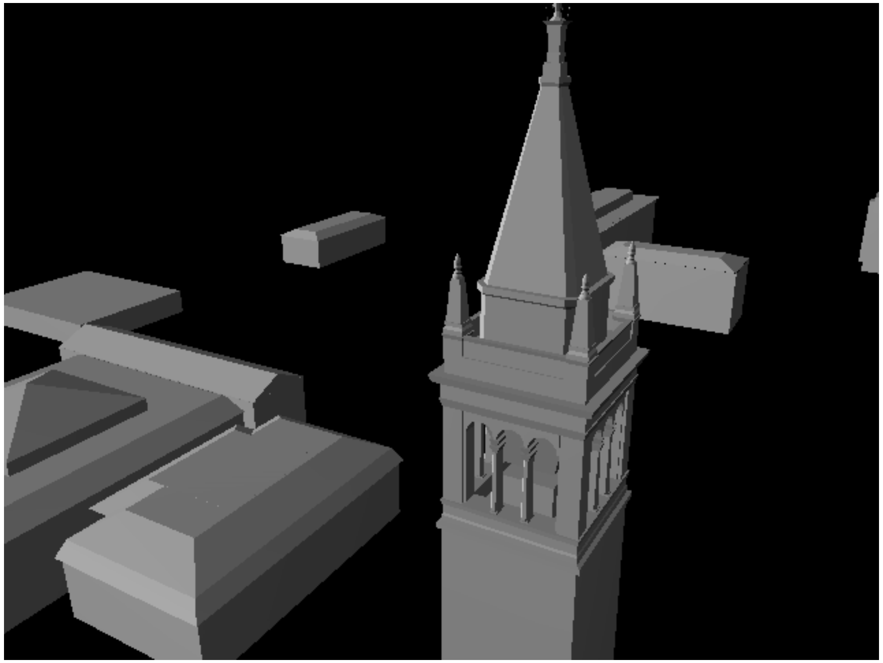




Photogrammetry

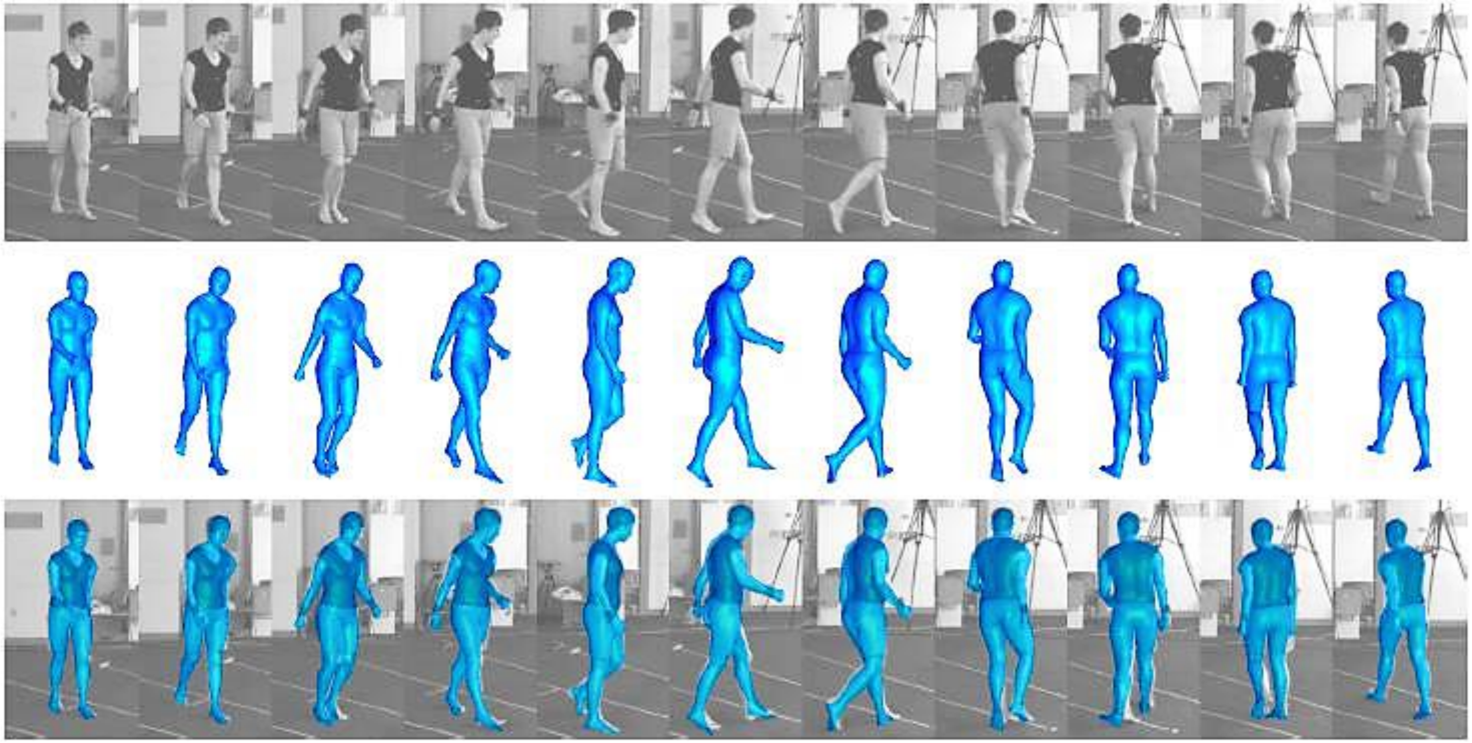
- Reconstruction from photographs





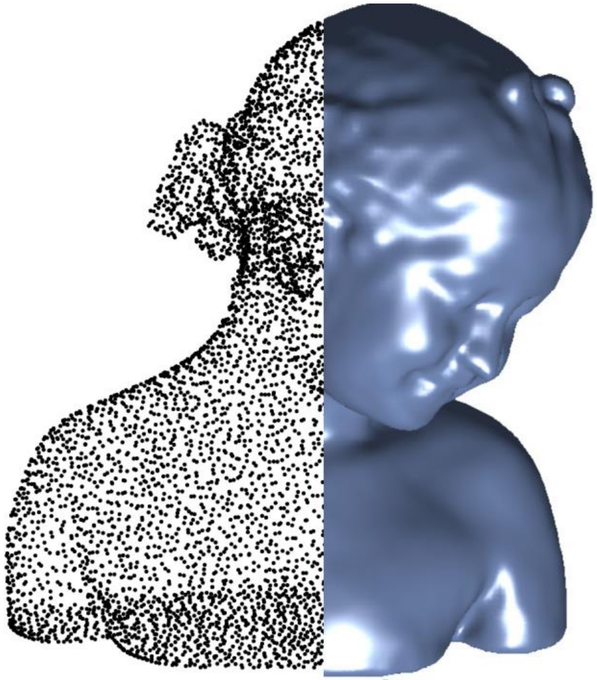
Photogrammetry

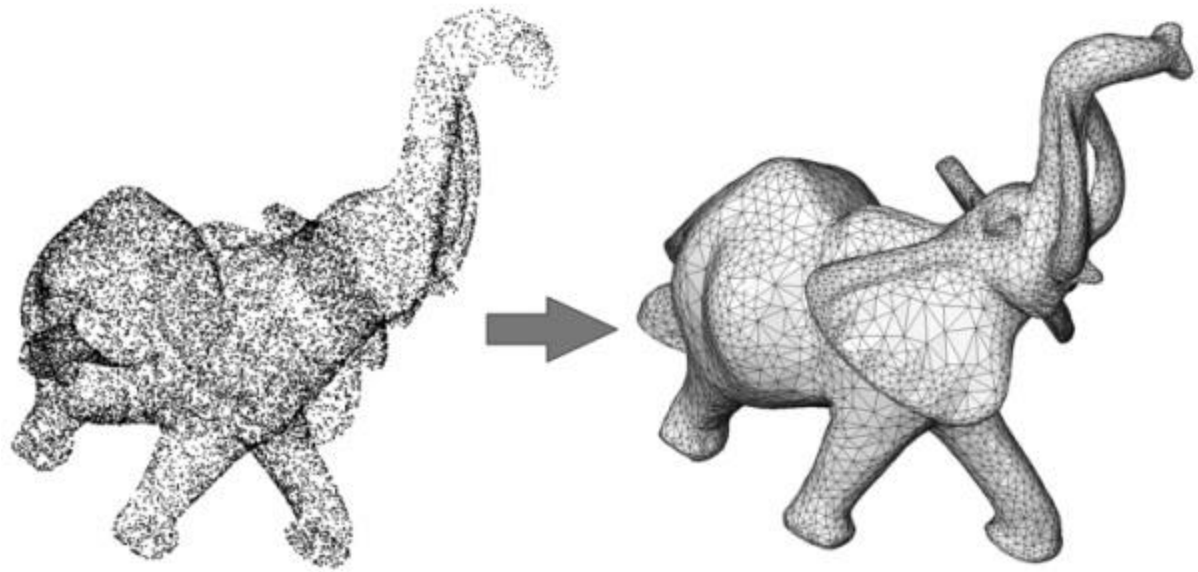
- Reconstruction from a series of photos (video)



Range Scanning

- Reconstruction from point cloud



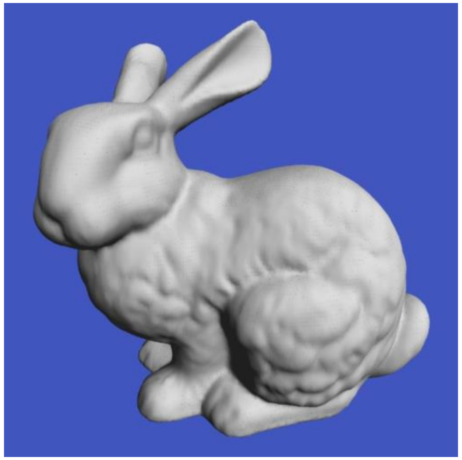


Getting Meshes from Real Objects

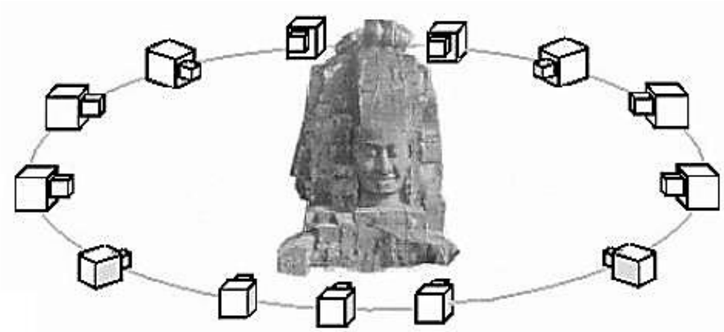
- Many models used in Graphics are obtained from real objects

Stanford dragon

- Faces： 871414
- Vertices： 437645
- Compressed： 8.2 MB in PLY format



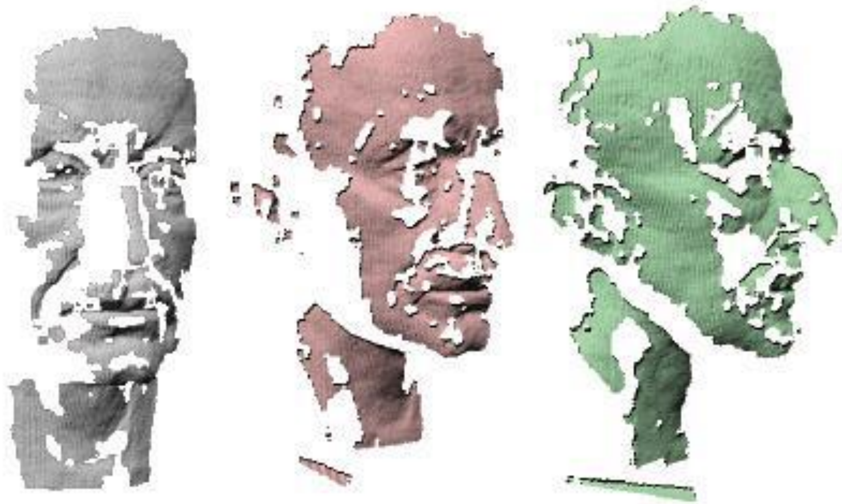
Getting Meshes from Real Objects



Range Scanning

- Accurate calibration is crucial
- Multiple scans required for complex objects
  - scan path planning
  - scan registration
- Scans are incomplete and noisy
  - model repair, hole filling
  - smoothing for noise removal

Range Scanning: Reconstruction



Set of raw scans

Reconstructed model

General Used Mesh Files

- General used mesh files
  - Wavefront OBJ (*.obj)
  - 3D Max (*.max, *.3ds)
  - VRML(*.vrl)
  - Inventor (*.iv)
  - PLY (*.ply, *.ply2)
  - User-defined(*.m, *.liu)
- Storage
  - Text – (Recommended)
  - Binary

Wavefront OBJ File Format

- Vertices
  - Start with char ‘v’
  - (x,y,z) coordinates
- Faces
  - Start with char ‘f’
  - Indices of its vertices in the file
- Other properties
  - Normal, texture coordinates, material, etc.

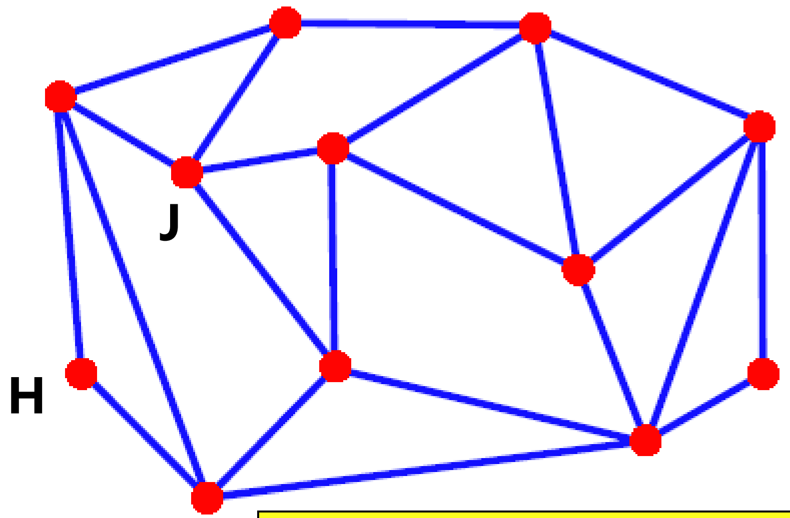
```
v  1.0  0.0  0.0
v  0.0  1.0  0.0
v  0.0 -1.0  0.0
v  0.0  0.0  1.0
f  1  2  3
f  1  4  2
f  3  2  4
f  1  3  4
```

Wavefront .obj file

```
# List of Vertices, with (x,y,z[,w]) coordinates, w is optional and defaults to 1.0.
  v 0.123 0.234 0.345 1.0
  v ...
  ...
  # Texture coordinates, in (u ,v [,w]) coordinates, these will vary between 0 and 1, w is optional and
default to 0.
  vt 0.500 1 [0]
  vt ...
  ...
  # Normals in (x,y,z) form; normals might not be unit.
.
  vn 0.707 0.000 0.707
  vn ...
  ...
  # Parameter space vertices in ( u [,v] [,w] ) form; free form geometry statement ( see below )
  vp 0.310000 3.210000 2.100000
  vp ...
  ...
  # Face Definitions (see below)
  f 1 2 3
  f 3/1 4/2 5/3
  f 6/4/1 3/5/3 7/6/5
  f ...
  ...
```

Meshes:
Definitions & Terminologies

Standard Graph Definition



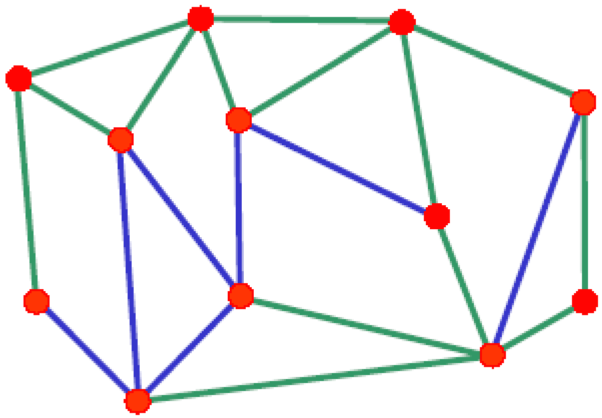
G = <**V**,**E**>
V = vertices =
{A,B,C,D,E,F,G,H,I,J,K,L}
E = edges =
{(A,B),(B,C),(C,D),(D,E),(E,F),(F,G),
(G,H),(H,A),(A,J),(A,G),(B,J),(K,F),
(C,L),(C,I),(D,I),(D,F),(F,I),(G,K),
(J,L),(J,K),(K,L),(L,I)}

Vertex degree (valence) = number of edges incident on vertex
deg(J) = 4, deg(H) = 2
k-regular graph = graph whose vertices all have degree *k*

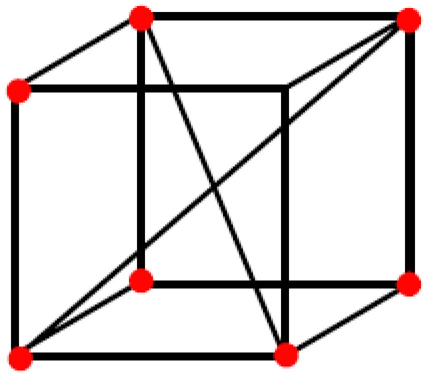
Face: cycle of vertices/edges which cannot be shortened
F = faces =
{(A,H,G),(A,J,K,G),(B,A,J),(B,C,L,J),(C,I,J),(C,D,I),
(D,E,F),(D,I,F),(L,I,F,K),(L,J,K),(K,F,G)}

Graph Embedding

Graph is ***embedded*** in R^d if each vertex is assigned a position in R^d



Embedding in R^2



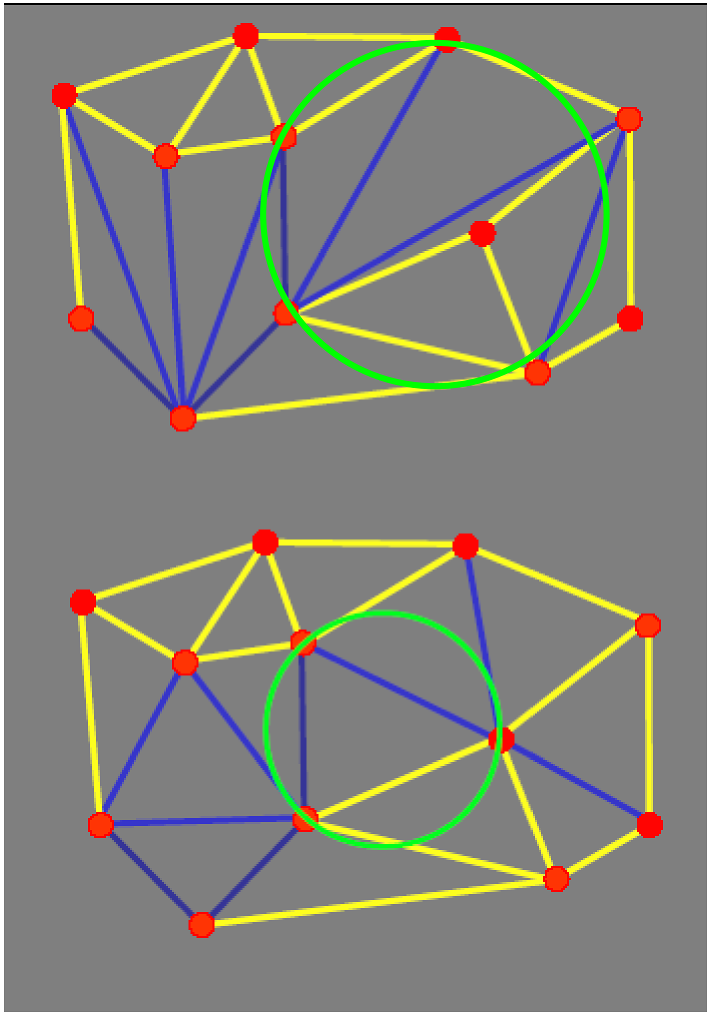
Embedding in R^3

Triangulation

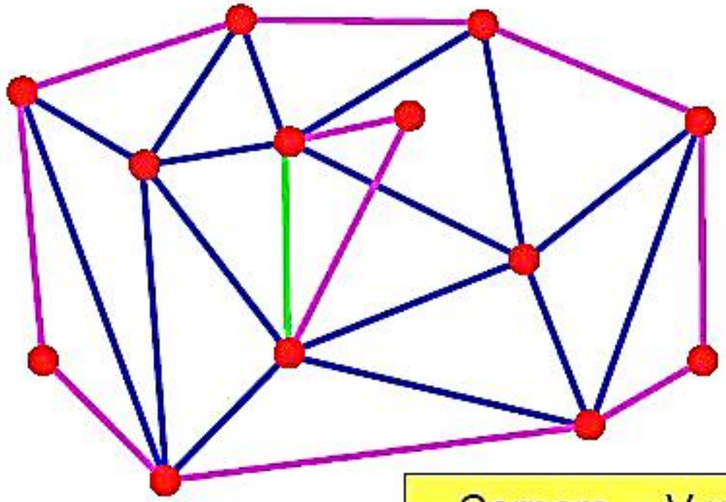
Triangulation: straight line plane graph all of whose faces are triangles

Delaunay triangulation of a set of points: unique set of triangles such that the circumcircle of any triangle does not contain any other point

Delaunay triangulation avoids long and skinny triangles



Meshes

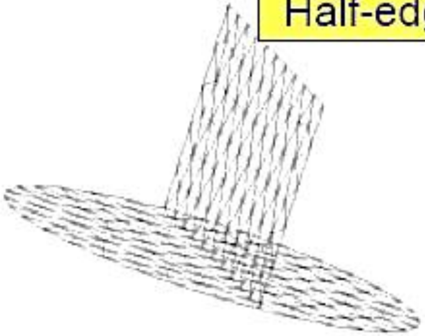


Mesh: straight-line graph embedded in R^3

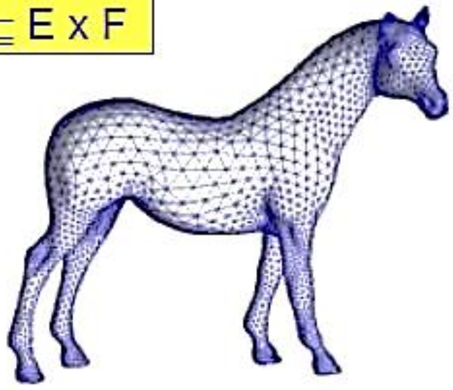
Boundary edge: adjacent to exactly *one* face
Regular edge: adjacent to exactly *two* faces
Singular edge: adjacent to more than two faces

Closed mesh: mesh with no boundary edges
Manifold mesh: mesh with no singular edges

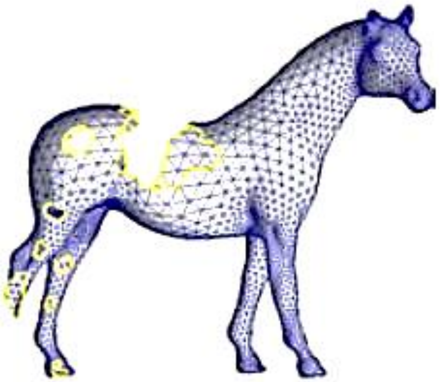
Corners $\subseteq$ V x F
Half-edges $\subseteq$ E x F



Non-Manifold

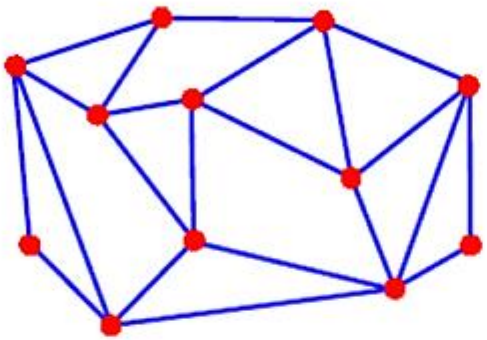


Closed Manifold



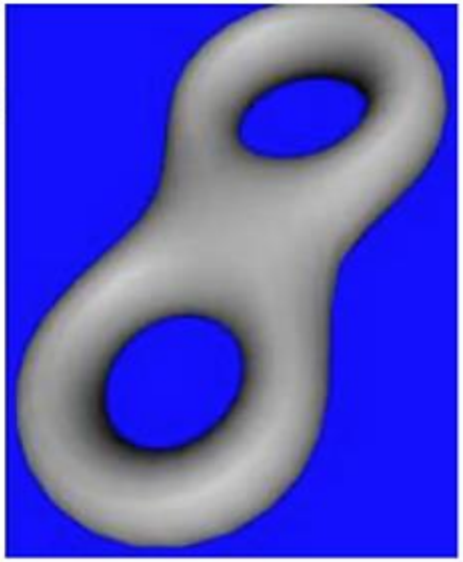
Open Manifold 8

Topology



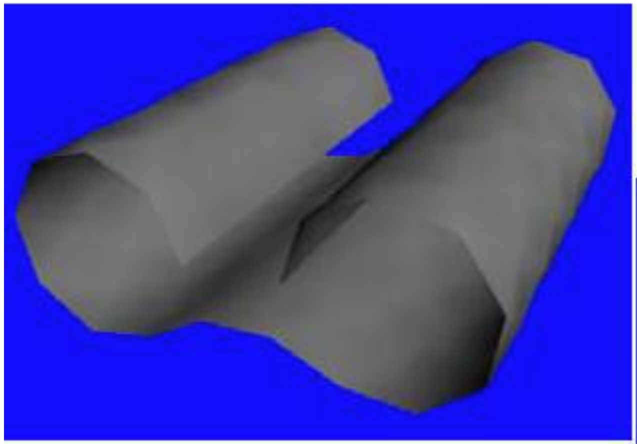
亏格 ***Genus*** of graph: *half* of *maximal* number of closed paths that do *not* disconnect the graph (number of “holes”)

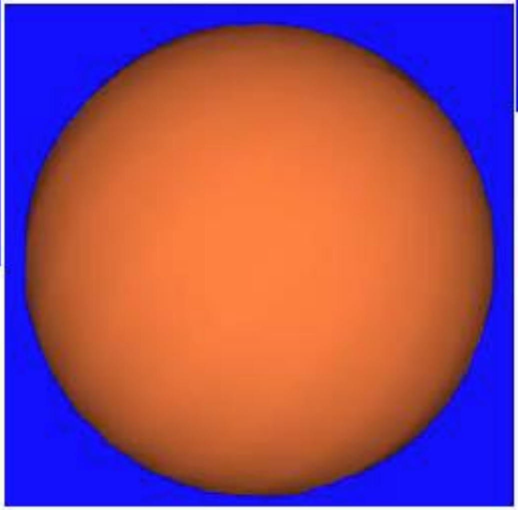
Genus(sphere)= 0
Genus(torus) = 1

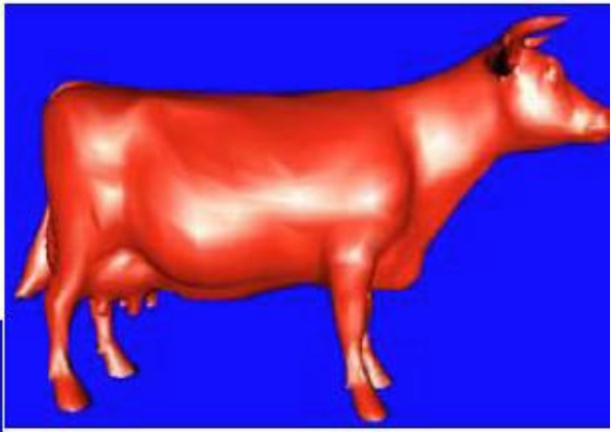


Developablity (可展性)

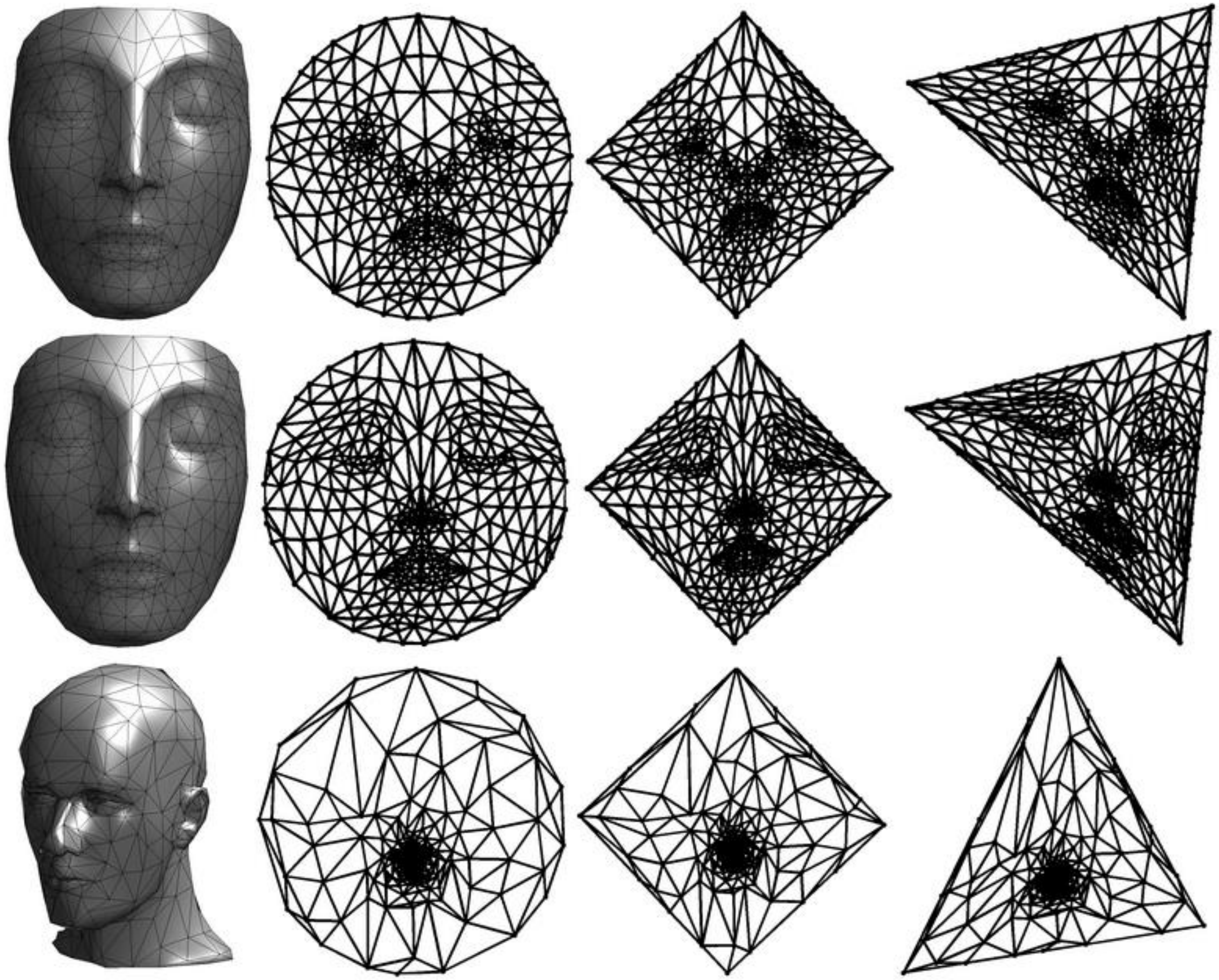
Mesh is ***developable*** if it may be embedded in R^2 without distortion







Developablity (可展性)

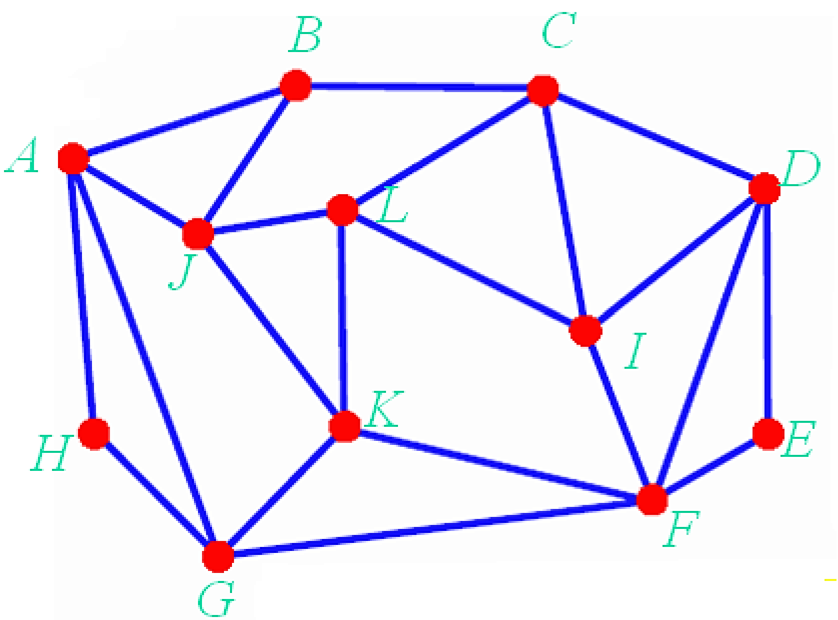


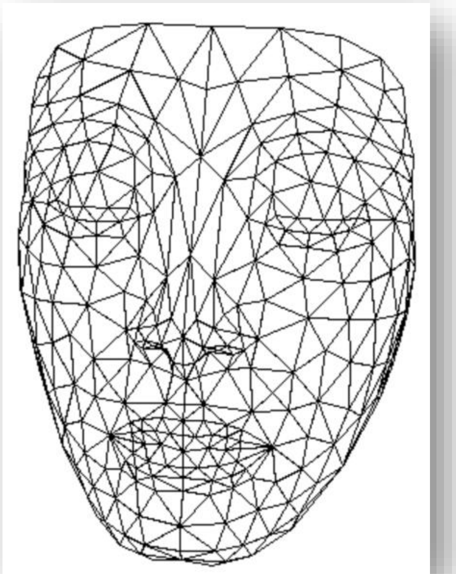
Mesh Data Structure

- How to store geometry and connectivity?
- Geometry queries
  - What are the vertices of face #k?
  - Are vertices #i and #j adjacent?
  - Which faces are adjacent face #k?
- Geometry operations
  - Remove/add a vertex/face
  - Mesh simplification
  - Vertex split, edge collapse

Define a mesh

- Geometry
  - Vertex coordinates
- Connectivity
  - How do vertices connected?



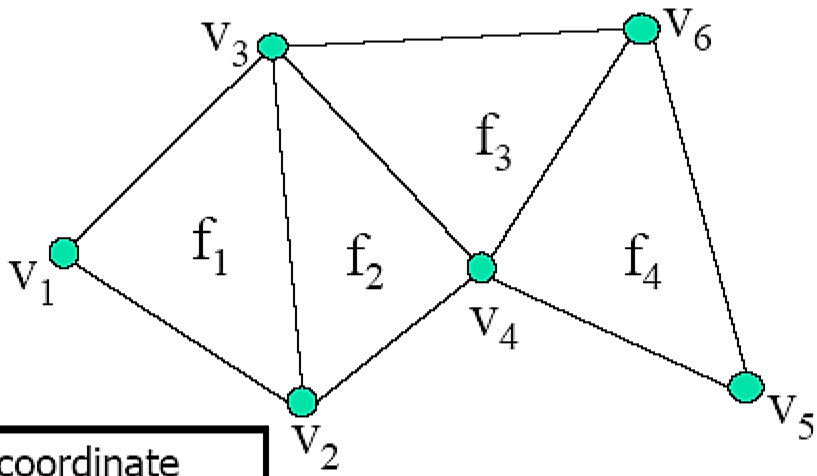


- List of Edge
- Vertex-Edge
- Vertex-Face
- Combined

List of Faces

- List of vertices
  - Position coordinates
- List of faces
  - Triplets of pointers to face vertices (c1,c2,c3)
- Queries:
  - What are the vertices of face #3?
    - Answered in O(1) - checking third triplet
  - Are vertices i and j adjacent?
    - A pass over all faces is necessary – NOT GOOD

List of Faces – Example



vertex	coordinate
v_1	(x_1,y_1,z_1)
v_2	(x_2,y_2,z_2)
v_3	(x_3,y_3,z_3)
v_4	(x_4,y_4,z_4)
v_5	(x_5,y_5,z_5)
v_6	(x_6,y_6,z_6)

face	vertices (ccw)
f_1	(v_1, v_2, v_3)
f_2	(v_2, v_4, v_3)
f_3	(v_3, v_4, v_6)
f_4	(v_4, v_5, v_6)

List of Faces – Analysis

- Pros:
  - Convenient and efficient (memory wise)
  - Can represent non-manifold meshes
- Cons:
  - Too simple - not enough information on relations between vertices & faces

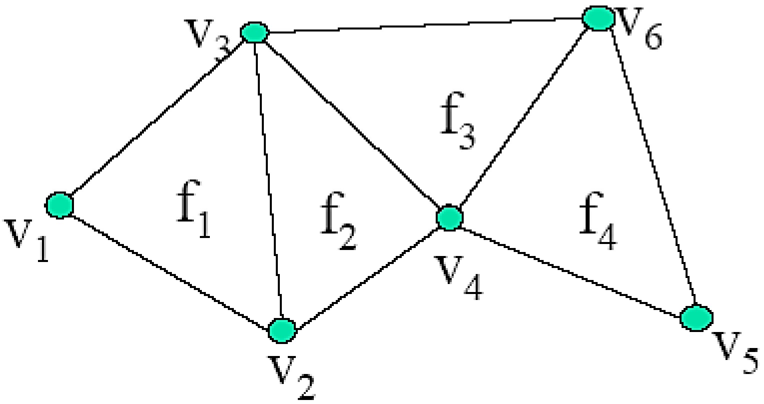
Adjacency Matrix – Definition

- View mesh as connected graph
- Given n vertices build n*n matrix of adjacency information
  - Entry (i,j) is TRUE value if vertices i and j are adjacent
- Geometric info
  - list of vertex coordinates
- Add faces
  - list of triplets of vertex indices (v1,v2,v3)

Adjacency Matrix – Example

vertex	coordinate
v_1	(x_1,y_1,z_1)
v_2	(x_2,y_2,z_2)
v_3	(x_3,y_3,z_3)
v_4	(x_4,y_4,z_4)
v_5	(x_5,y_5,z_5)
v_6	(x_6,y_6,z_6)

face	vertices (ccw)
f_1	(v_1, v_2, v_3)
f_2	(v_2, v_4, v_3)
f_3	(v_3, v_4, v_6)
f_4	(v_4, v_5, v_6)



	v_1	v_2	v_3	v_4	v_5	v_6
v_1		1	1			
v_2	1		1	1		
v_3	1	1		1		1
v_4		1	1		1	1
v_5				1		1
v_6			1	1	1	

Adjacency Matrix – Queries

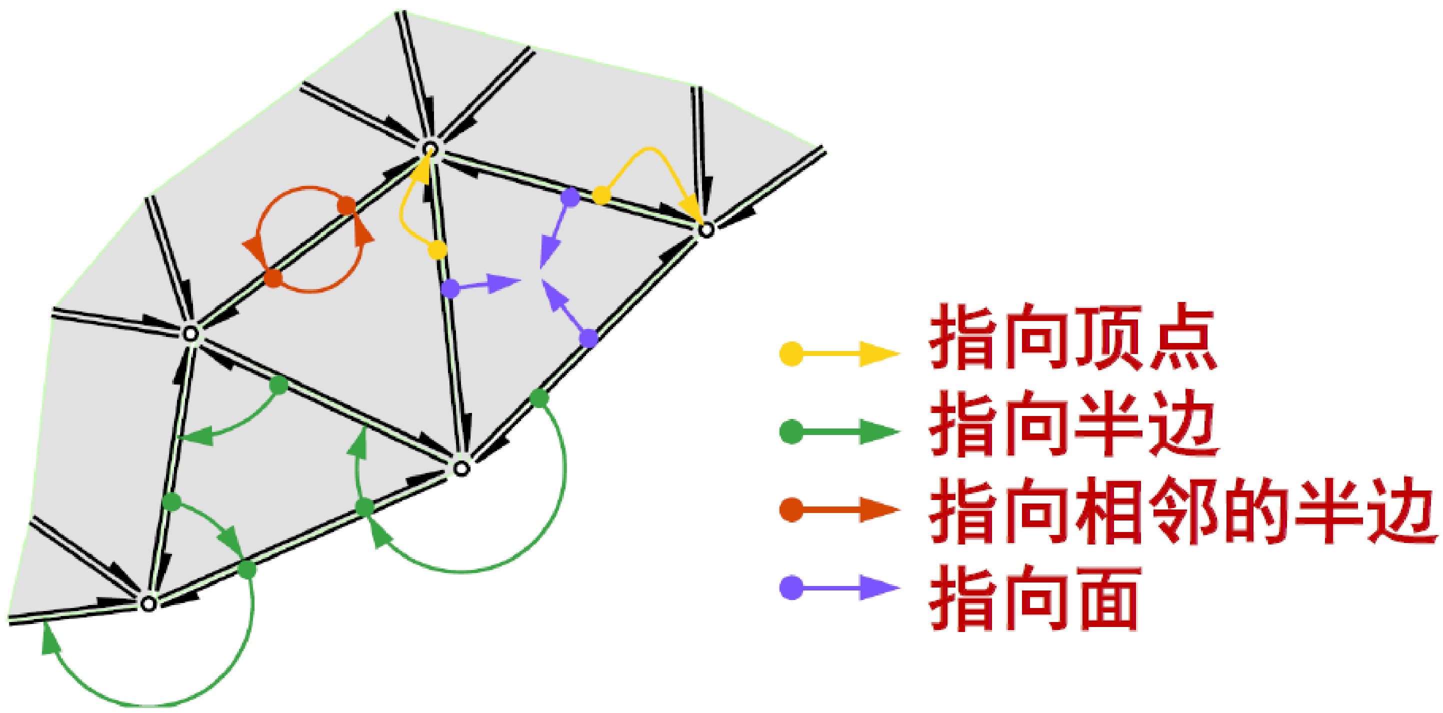
- What are the vertices of face #3?
  - O(1) – checking third triplet of faces
- Are vertices i and j adjacent?
  - O(1) - checking adjacency matrix at location (i,j).
- Which faces are adjacent to vertex j?
  - Full pass on all faces is necessary

Adjacency Matrix – Analysis

- Pros:
  - Information on vertices adjacency
  - Stores non-manifold meshes
- Cons:
  - Connects faces to their vertices, BUT NO connection between vertex and its face

Half-Edge Structure

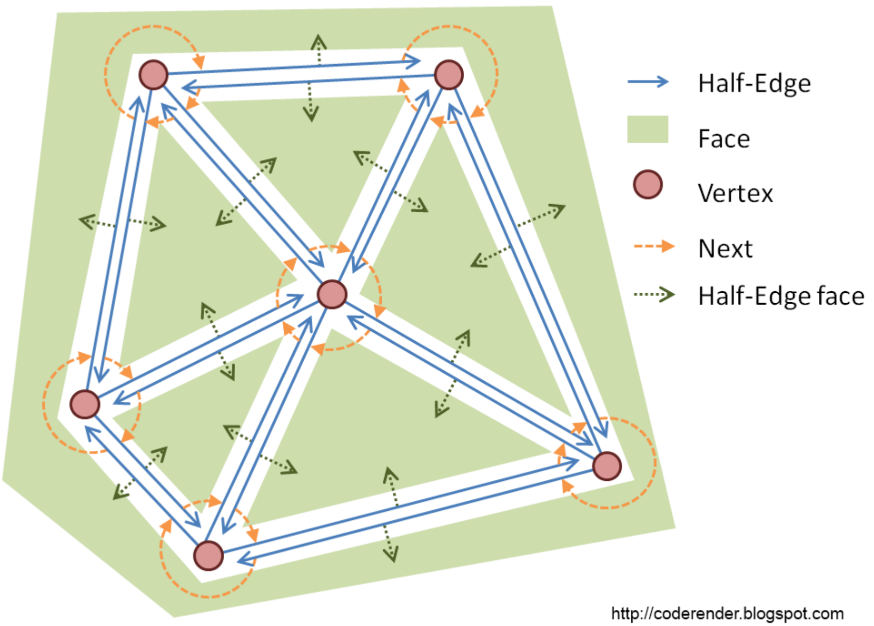
- Orientable 2D manifolds and its sub set: special polygonal meshes (适用于有向的二维流形)



Half-Edge Structure

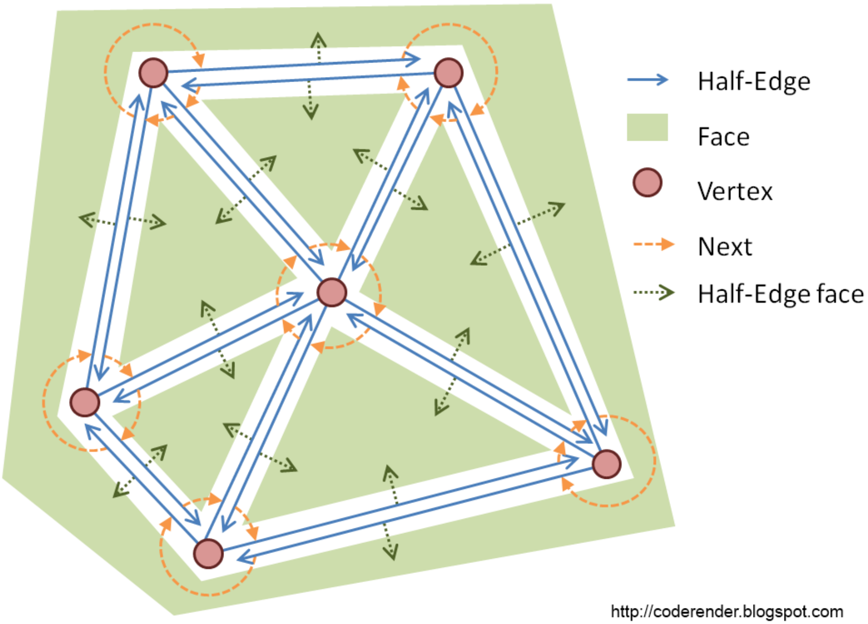
- Half-edge (each edge corresponds to two half-edges)
    - Pointer to the first vertices
    - To adjacent face
    - To next half-edge (逆时针方向)
    - To the other half-edge of the same edge
    - To previous half-edge (opt.)

```
struct HE_edge {
    HE_vert* vert; // vertex at the start of the half-edge
    HE_face* face; // face the half-edge borders
    HE_edge* pair; // oppositely oriented adjacent half-edge
    HE_edge* next; // next half-edge around the face
    HE_edge* prev; // prev half-edge around the face
};
```



Half-Edge Structure

- Face： we only need a pointer to one of its half-edge



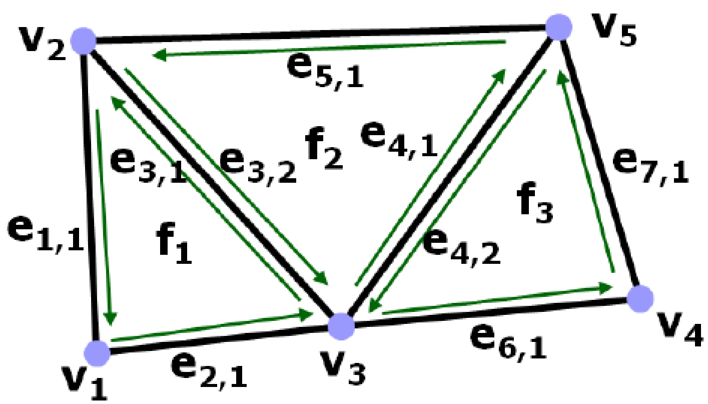

```
struct HE_face {
    HE_edge* edge; // one of the half-edges bordering the face
};
```

Half-Edge Structure

- Vertices
  - 3D coordinates
  - Pointer to the half-edge starting from it

```
struct HE_vert {
    float x;
    float y;
    float z;
    HE_edge* edge; // one of the half-edges
                   //emantating from the vertex
};
```

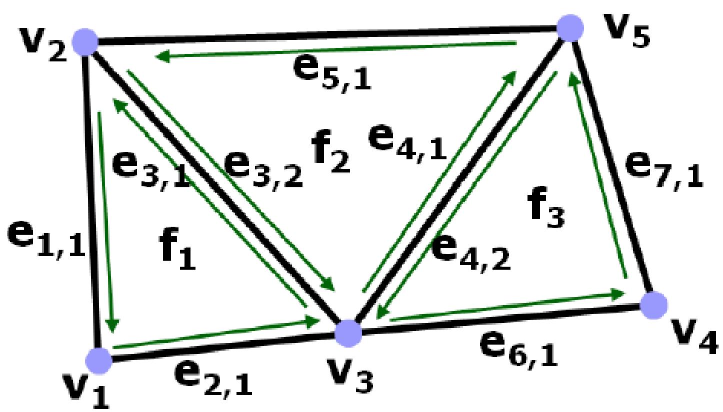
Example: half-edge structure



顶点	坐标	以此为起点的半边
v_1	(x_1,y_1,z_1)	$e_{2,1}$
v_2	(x_2,y_2,z_2)	$e_{1,1}$
v_3	(x_3,y_3,z_3)	$e_{4,1}$
v_4	(x_4,y_4,z_4)	$e_{7,1}$
v_5	(x_5,y_5,z_5)	$e_{5,1}$

面	半边
f_1	$e_{1,1}$
f_2	$e_{3,2}$
f_3	$e_{4,2}$

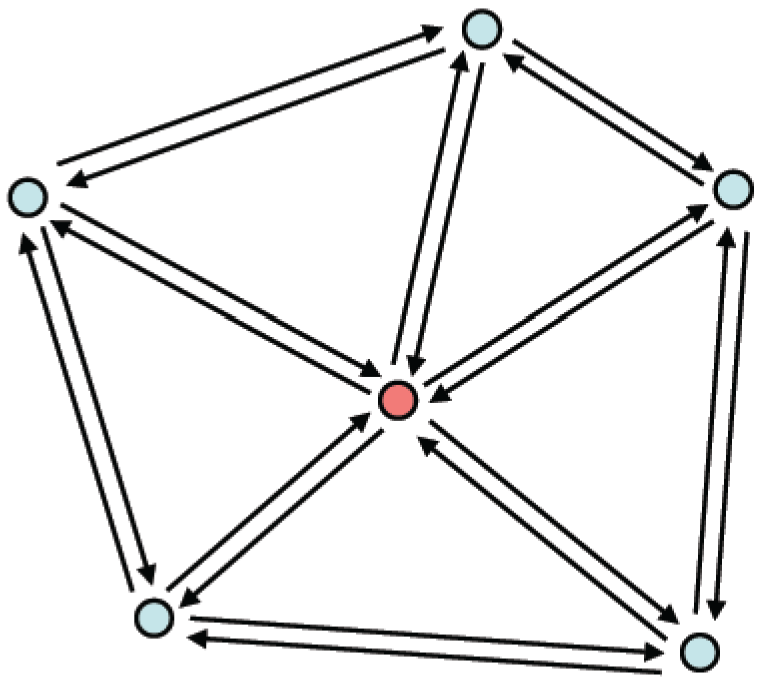
Example (continued)



半边	起点	相邻半边	面	下条半边	前条半边
$e_{3,1}$	v_3	$e_{3,2}$	f_1	$e_{1,1}$	$e_{2,1}$
$e_{3,2}$	v_2	$e_{3,1}$	f_2	$e_{4,1}$	$e_{5,1}$
$e_{4,1}$	v_3	$e_{4,2}$	f_2	$e_{5,1}$	$e_{3,2}$
$e_{4,2}$	v_5	$e_{4,1}$	f_3	$e_{6,1}$	$e_{7,1}$

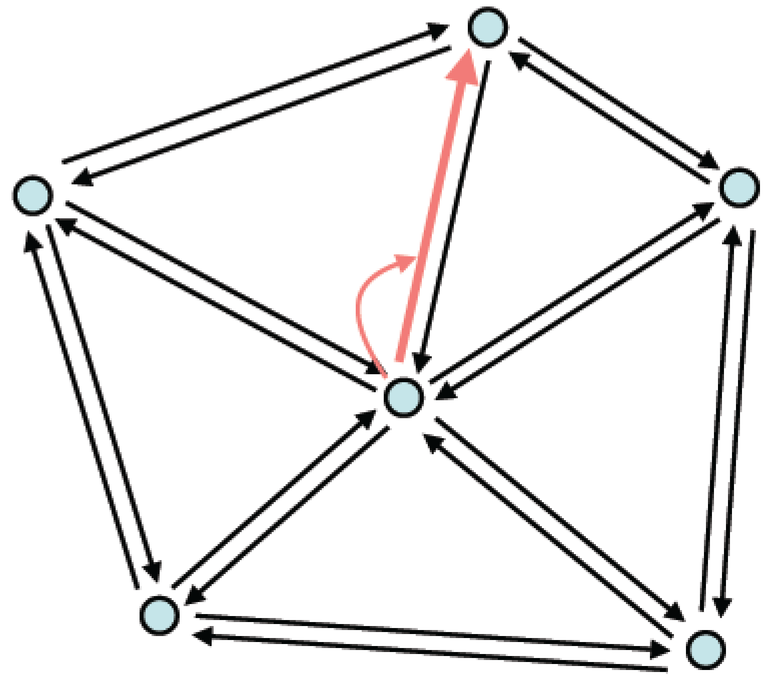
One-Ring Traversal

1. Start at vertex



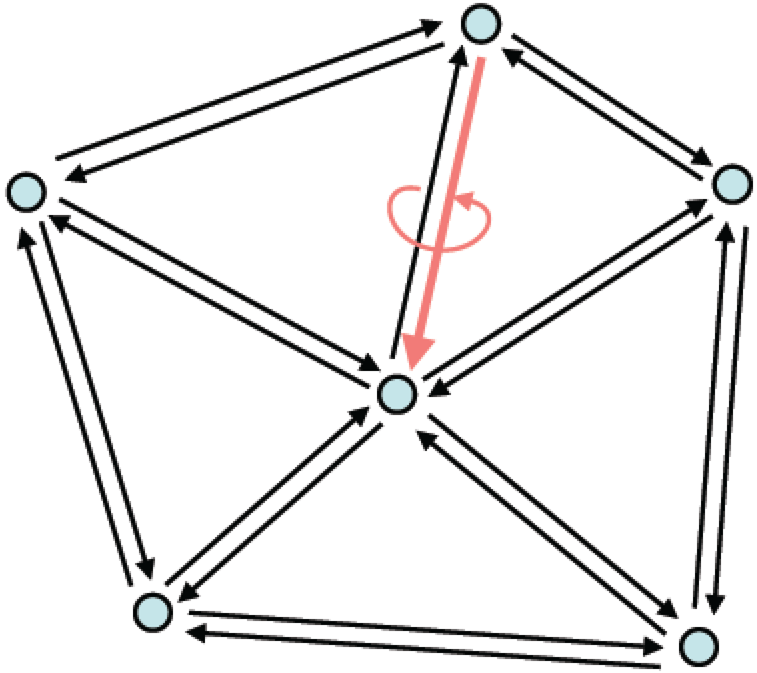
One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge



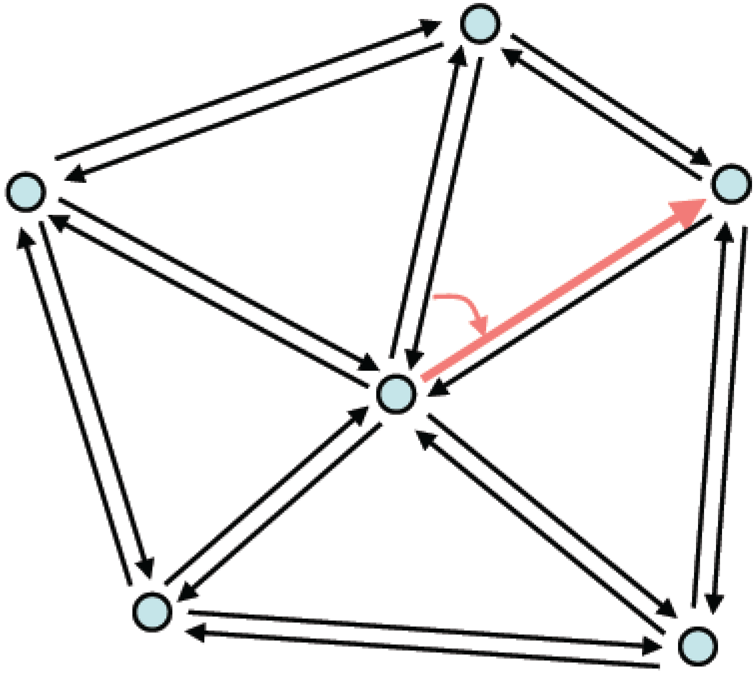
One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge
3. Opposite halfedge



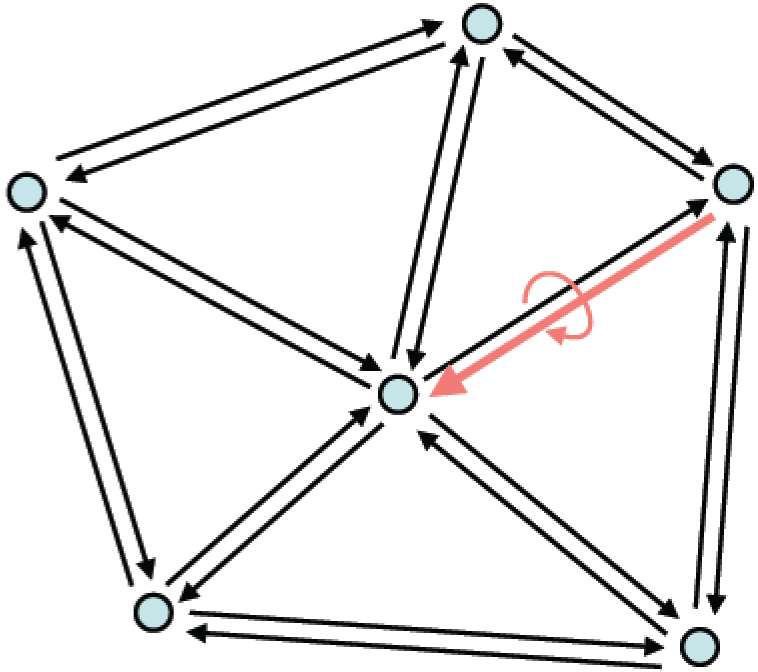
One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge
3. Opposite halfedge
4. Next halfedge



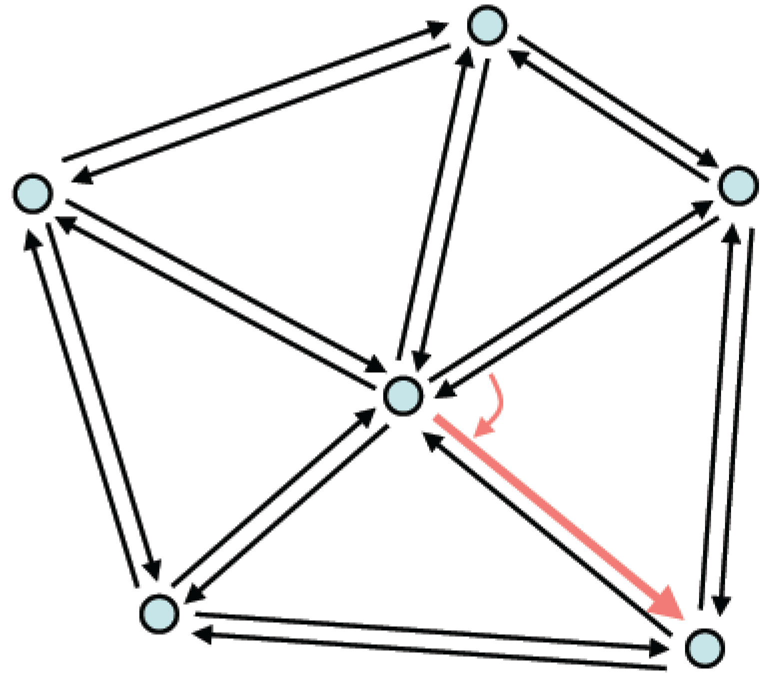
One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge
3. Opposite halfedge
4. Next halfedge
5. Opposite



One-Ring Traversal

1. Start at vertex
2. Outgoing halfedge
3. Opposite halfedge
4. Next halfedge
5. Opposite
6. Next
7. ...



Traversal operations

Vertices adjacent to a vertex v, mesh without boundary

```
he = v->halfedge;
 do {
  he = he->sym->next;
  ... // perform operations with
      // he->vertex
 } while (he != v->halfedge)
```

No “if” statements.

Basic operations

- Mark mesh boundary (标记边界点)
- Create edge adjacency (创建邻接边)
- Add vertex (增加顶点)
- Add edge (增加边)
- Add polygonal face (增加面)
- Delete polygonal face (删除面)
- Delete edge (删除边)
- Delete vertex (删除顶点)

Discussion

- Advantage and disadvantage(优缺点)：
  - Adv.：Query time O(1)，operation time O(1)
  - Dis.： redundancy & only applicable to 2D manifolds
- For more information refer to
  - CGAL：
    - the Computational Geometry Algorithms Library， http://www.cgal.org/
    - Free for non-commercial use
  - OpenMesh： http://www.openmesh.org/
    - Mesh processing
    - Free, LGPL licence
  - Meshlab: http://meshlab.sourceforge.net/

Advantage and disadvantage in polygon representation

- Advantage
  - Simplicity - ease of description
  - Based data for rendering software/hardware
  - Input to most simulation/analysis tools
  - Output of most acquisition tools
  - laser scanner, CT, MRI, etc...

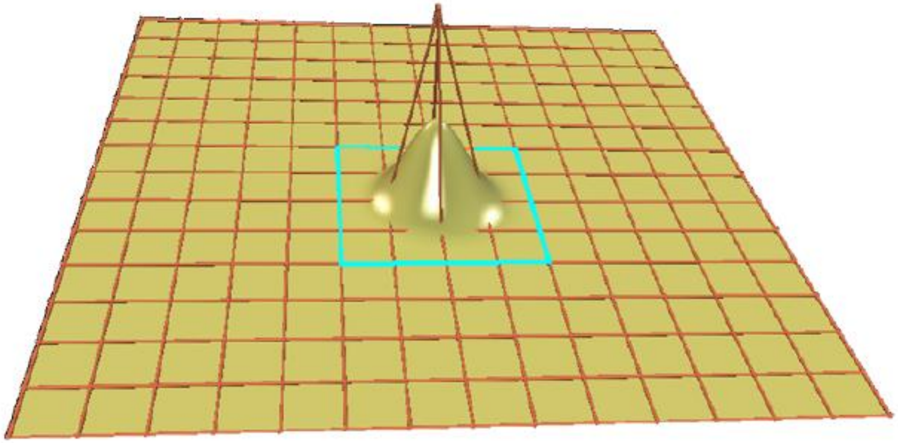
Advantage and disadvantage in polygon representation

- Disadvantage
  - Approximation, it is hard to satisfy real time interaction
  - It is hard to edit mesh with traditional method.
  - Without analytical form, geometric attribute is hard to compute
  - When expressed object with complex topology and rich details, modeling/editing/rendering/storing will have more burden.

Spline Surfaces

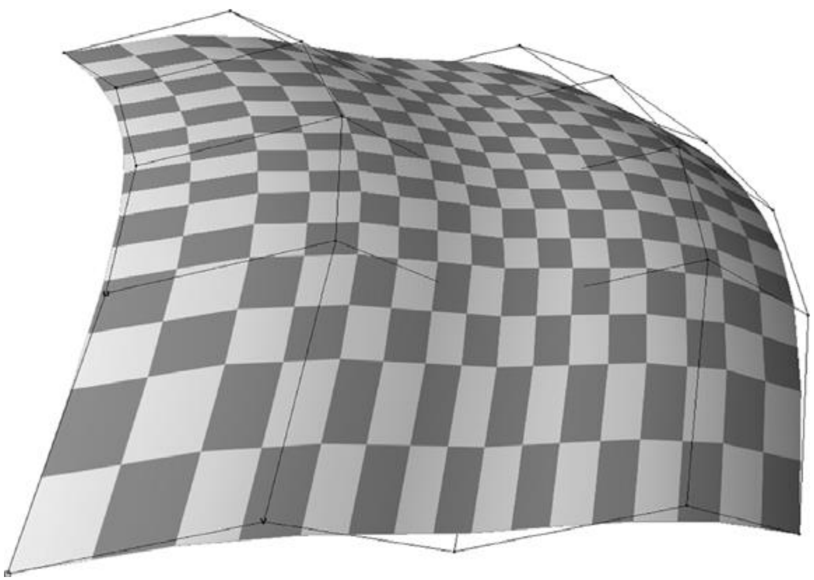
- Tensor product surfaces (“curves of curves”)
  - Rectangular grid of control points

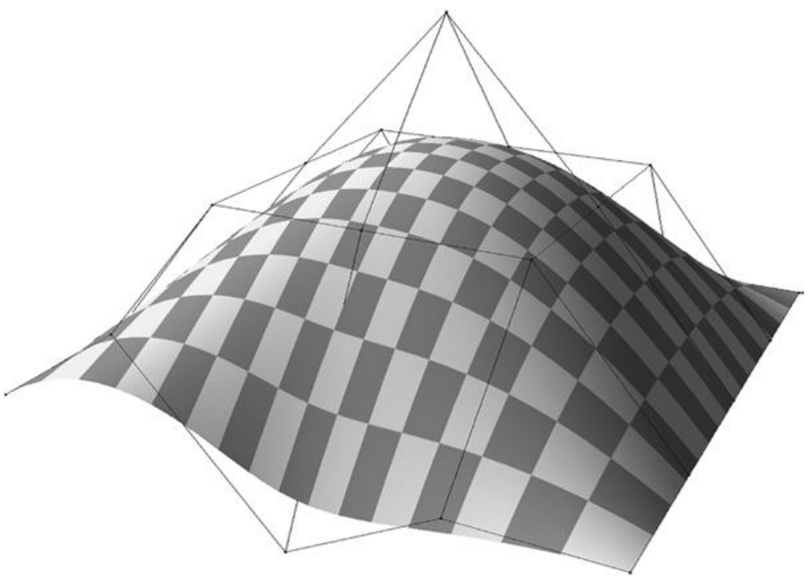
$$\mathbf{p}(u,v) = \sum_{i=0}^{k} \sum_{j=0}^{l} \mathbf{p}_{ij} \, N_i^n(u) N_j^n(v)$$



Spline Surfaces

- Tensor product surfaces ("curves of curves")
  - Rectangular grid of control points
  - Rectangular surface patch





Spline Surfaces

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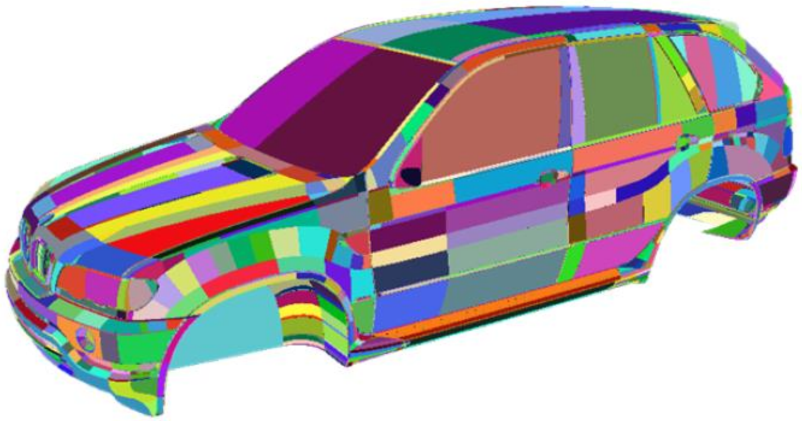
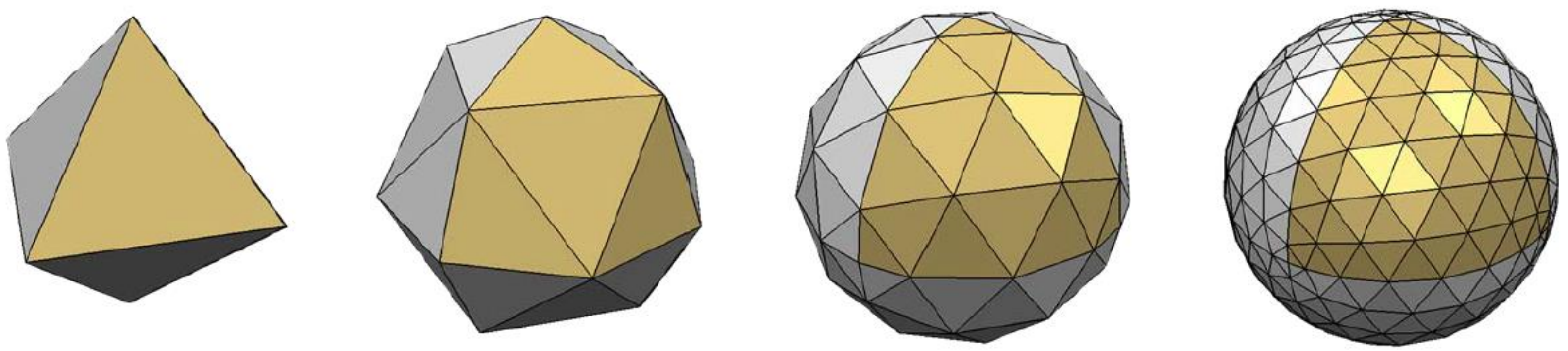


- Problems:
  - Many patches for complex models
  - Smoothness across patch boundaries
  - Trimming for non-rectangular patches

Subdivision Surfaces

- Generalization of spline curves/surfaces
  - Arbitrary control meshes
  - Successive refinement(subdivision)
  - Converges to Smooth limit surface
  - Connection between splines and meshes



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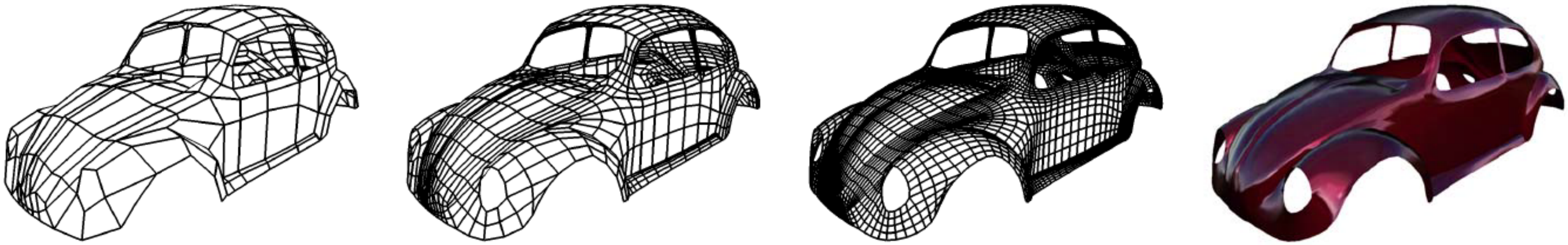
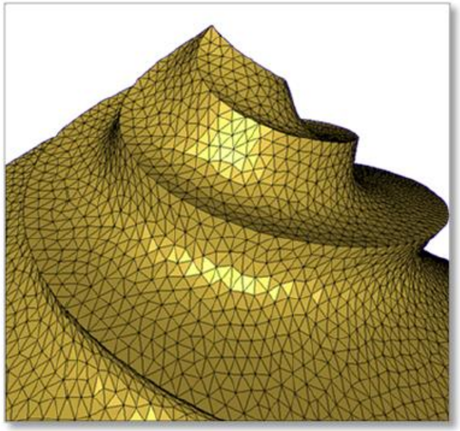


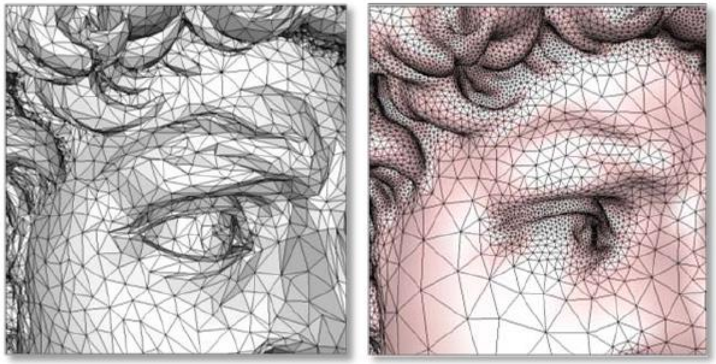
Discrete Surfaces: Point Sets, Meshes

- Flexible
- Suitable for highly detailed scanned data



- No analytic surface
- No inherent "editability"

Mesh editing

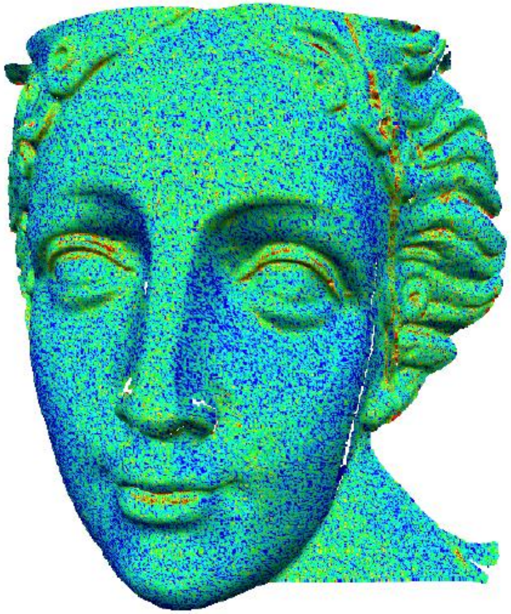


Mesh Processing & Editing

Mesh Denoising

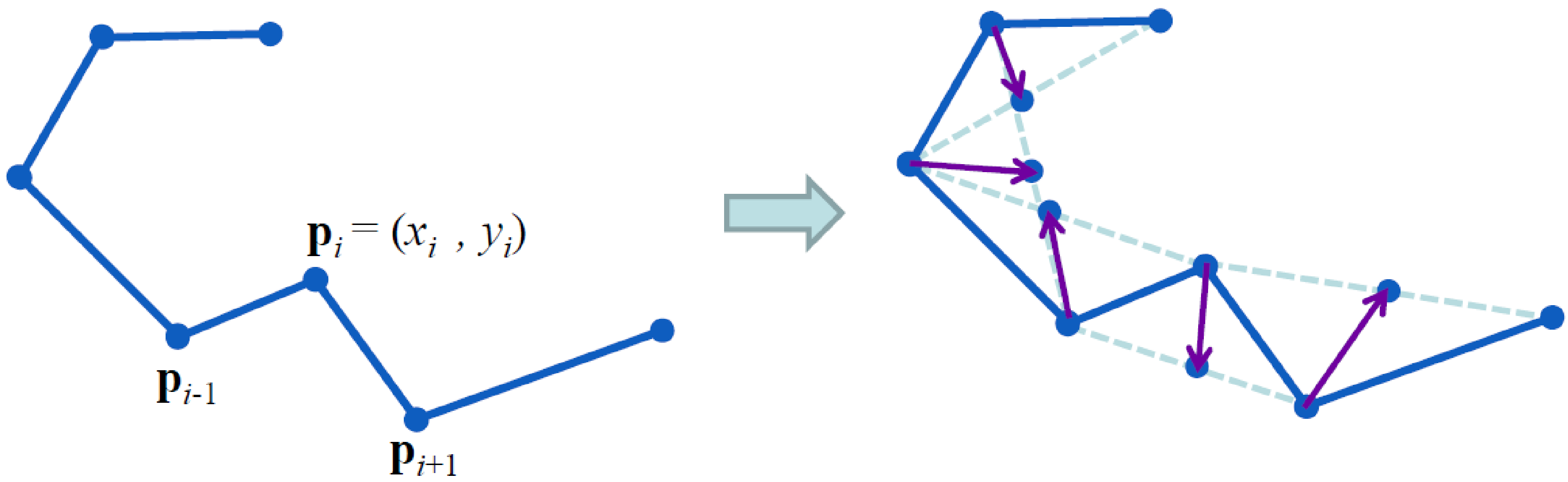
- Mesh Denoising (aka Smoothing, Filtering, Fairing)

Input: Noisy mesh (scanned or other)
Output: Smooth mesh
How: Filter out high frequency noise



Laplacian Smoothing

- An easier problem: How to smooth a curve?

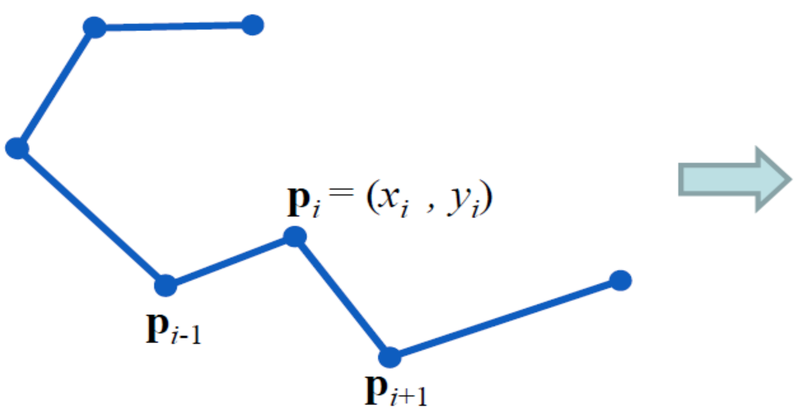


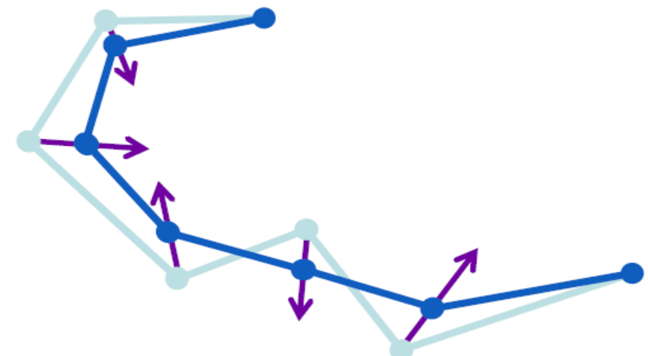
$$(\mathbf{p}_{i-1} + \mathbf{p}_{i+1})/2 - \mathbf{p}_i$$

$$L(\mathbf{p}_i) = \frac{1}{2}(\mathbf{p}_{i+1} - \mathbf{p}_i) + \frac{1}{2}(\mathbf{p}_{i-1} - \mathbf{p}_i)$$

Laplacian Smoothing

An easier problem: How to smooth a curve?





$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \frac{1}{2} L(\mathbf{p}_i)$$

Finite difference discretization of second derivative = Laplace operator in one dimension

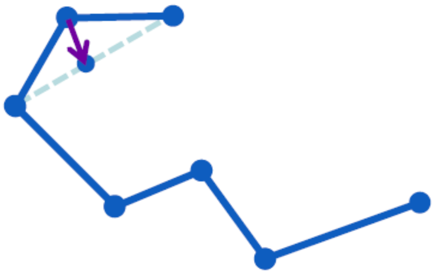
$$L(\mathbf{p}_i) = \frac{1}{2}(\mathbf{p}_{i+1} - \mathbf{p}_i) + \frac{1}{2}(\mathbf{p}_{i-1} - \mathbf{p}_i)$$

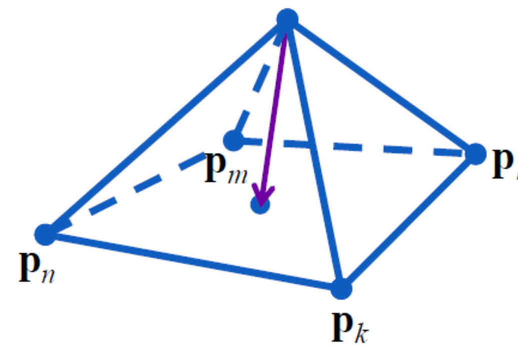
Laplacian Smoothing on Meshes

Same as for curves:

$$\mathbf{p}_i^{(t+1)} = \mathbf{p}_i^{(t)} + \lambda \Delta \mathbf{p}_i^{(t)}$$

What is $\Delta \mathbf{p}_i$?

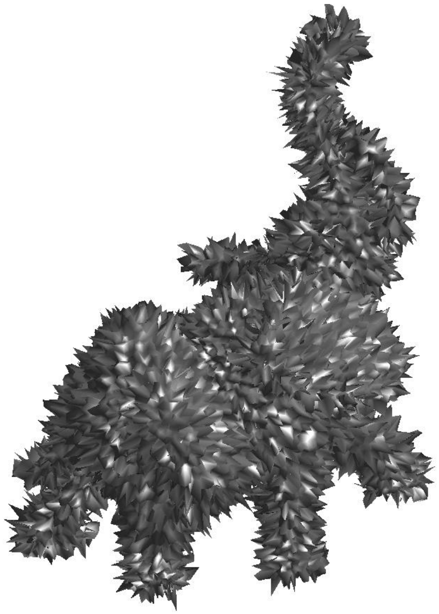




$$\frac{1}{2}(\mathbf{p}_{i+1} + \mathbf{p}_{i-1}) - \mathbf{p}_i \qquad \frac{1}{|N_i|}\left(\sum_{j \in N_i} \mathbf{p}_j\right) - \mathbf{p}_i$$

Mesh Denoising

- We generate artificially a noisy mesh by random normal displacement along the normal.



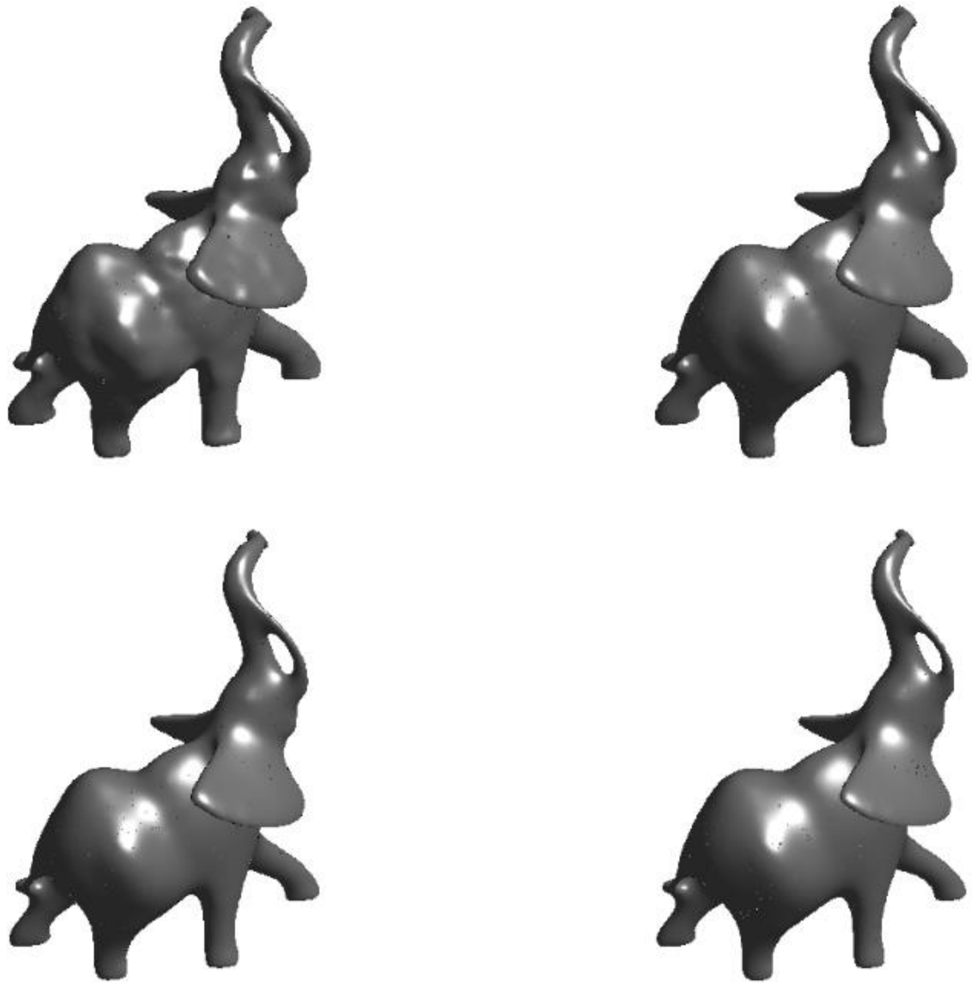
Mesh Denoising with Filtering

The quality of a noisy mesh is improved by applying local averagings, that removes noise but also tends to smooth features.

The operator $\tilde{W} : \mathbb{R}^n \to \mathbb{R}^n$ can be used to smooth a function, but it can also be applied to smooth the position $W \in \mathbb{R}^{3\times n}$. Since they are stored as row of a matrix, one should applies $\tilde{W}^*$ (transposed matrix) on the right side.

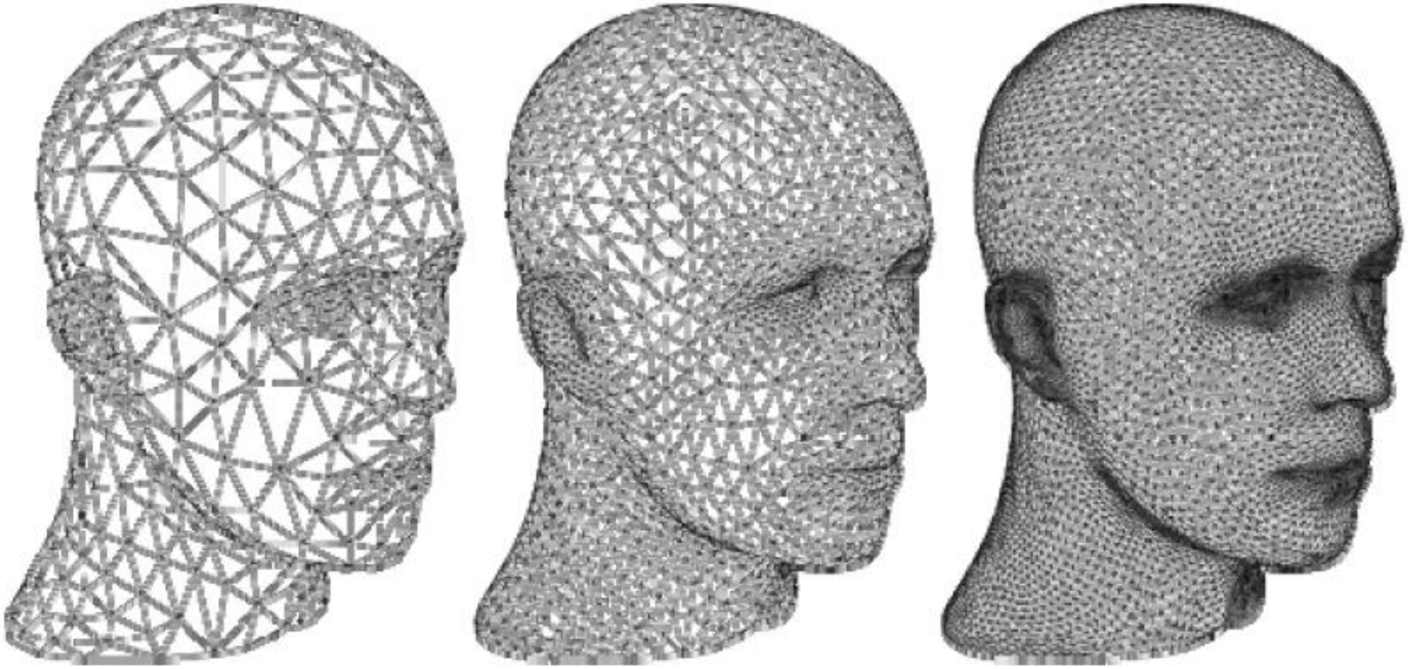
$$X^{(0)} = X \quad \text{and} \quad X^{(\ell+1)} = X^{(\ell)} W^*$$

Mesh Denoising with Filtering



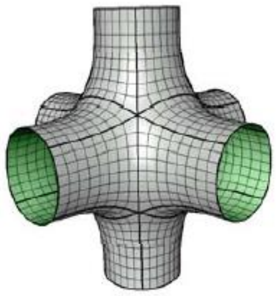
Mesh Subdivision

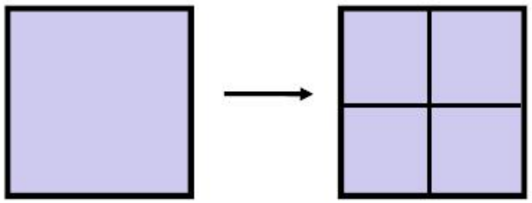
- No regular structure as for curves
  - Arbitrary number of edge-neighbors
  - Different subdivision rules for each valence



Subdivision Rules

- How the connectivity changes





- How the geometry changes
  - Old points
  - New points

Subdivision Zoo

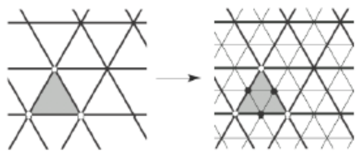
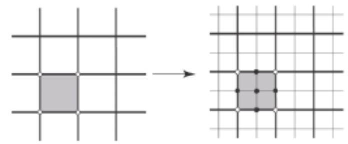
- Classification of subdivision schemes

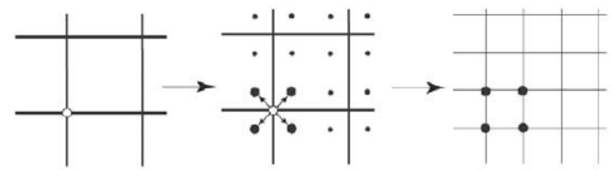
Primal	Faces are split into sub-faces
Dual	Vertices are split into multiple vertices

Approximating	Control points are not interpolated
Interpolating	Control points are interpolated

Subdivision Zoo

- Classification of subdivision schemes

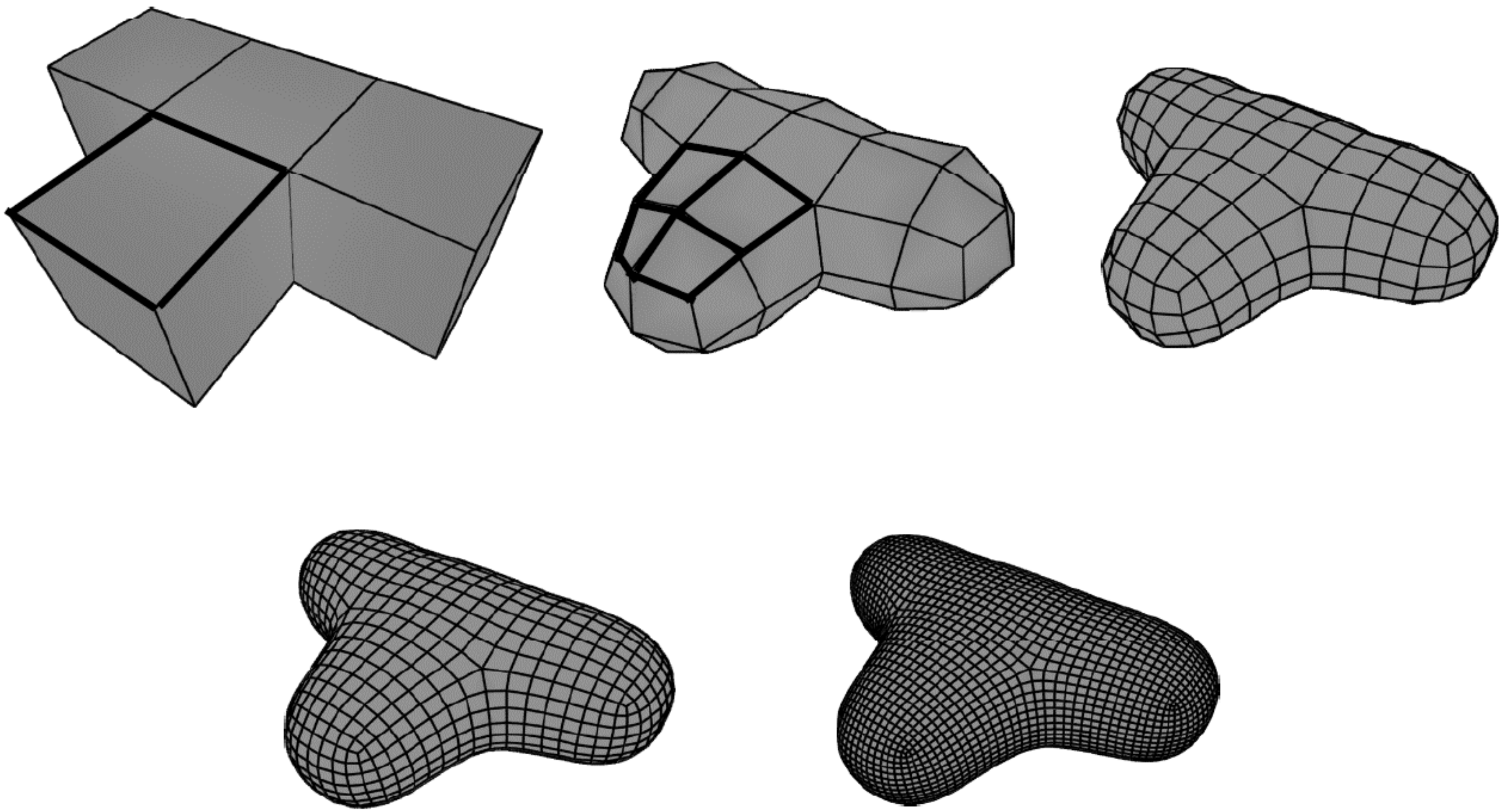
Primal (face split)		
		
	Triangular meshes	*Quad Meshes*
Approximating	Loop(C^2)	Catmull-Clark(C^2)
Interpolating	Mod. Butterfly (C^1)	Kobbelt (C^1)



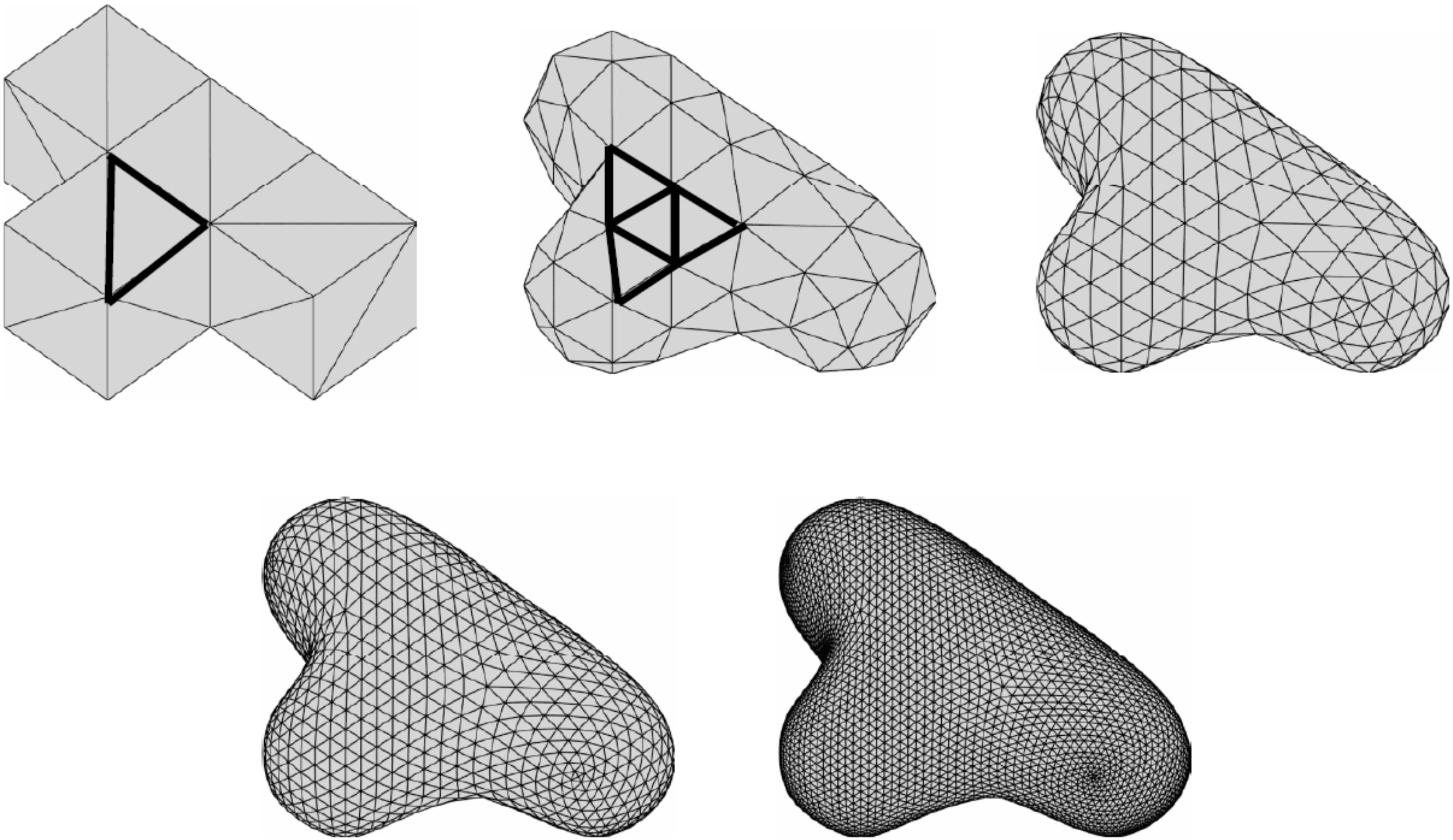
Dual (vertex split)
Doo-Sabin, Midedge(C^1)
Biquartic (C^2)

- Many more...

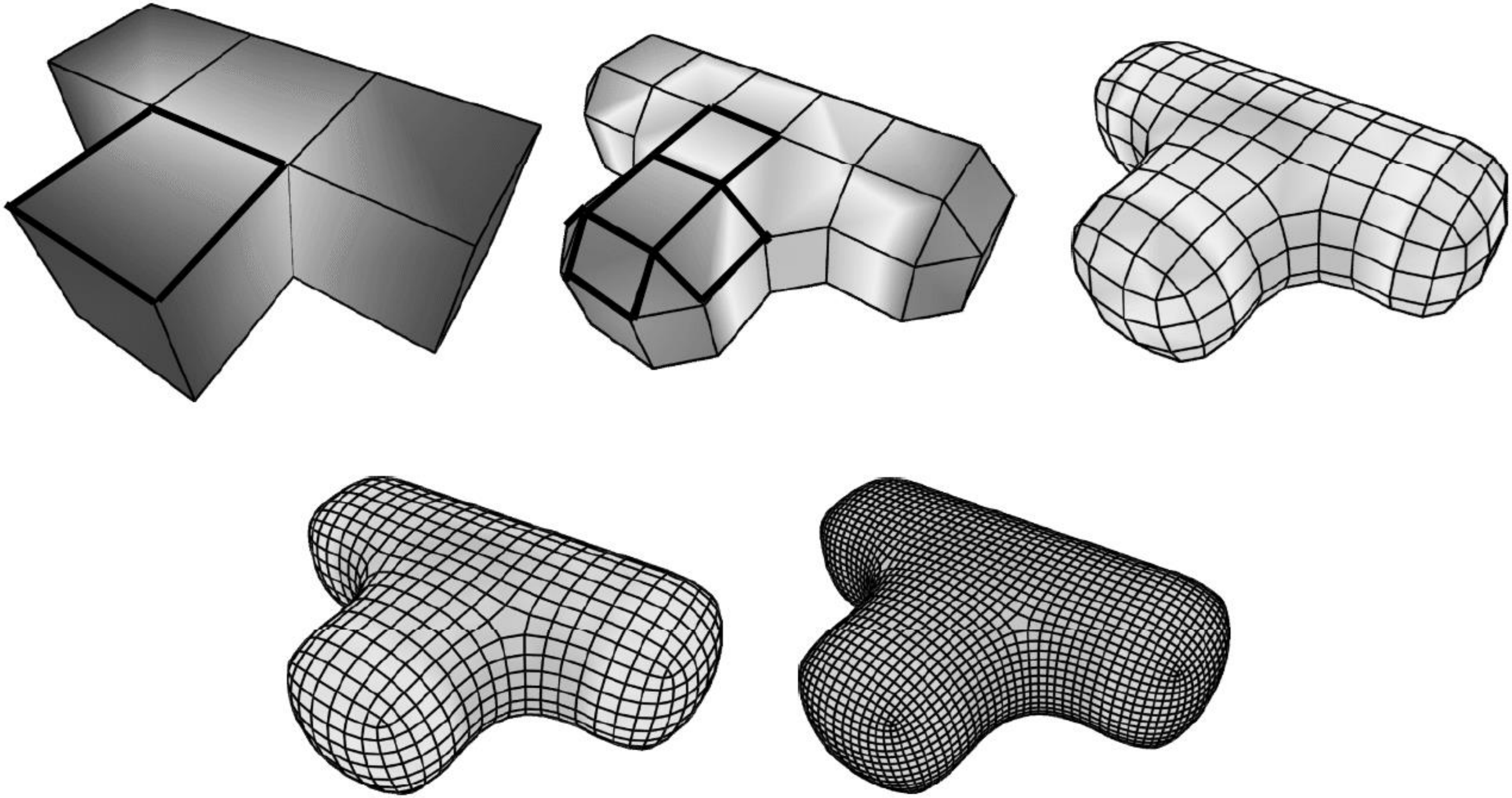
Catmull-Clark Subdivision



Loop Subdivision



Doo-Sabin Subdivision

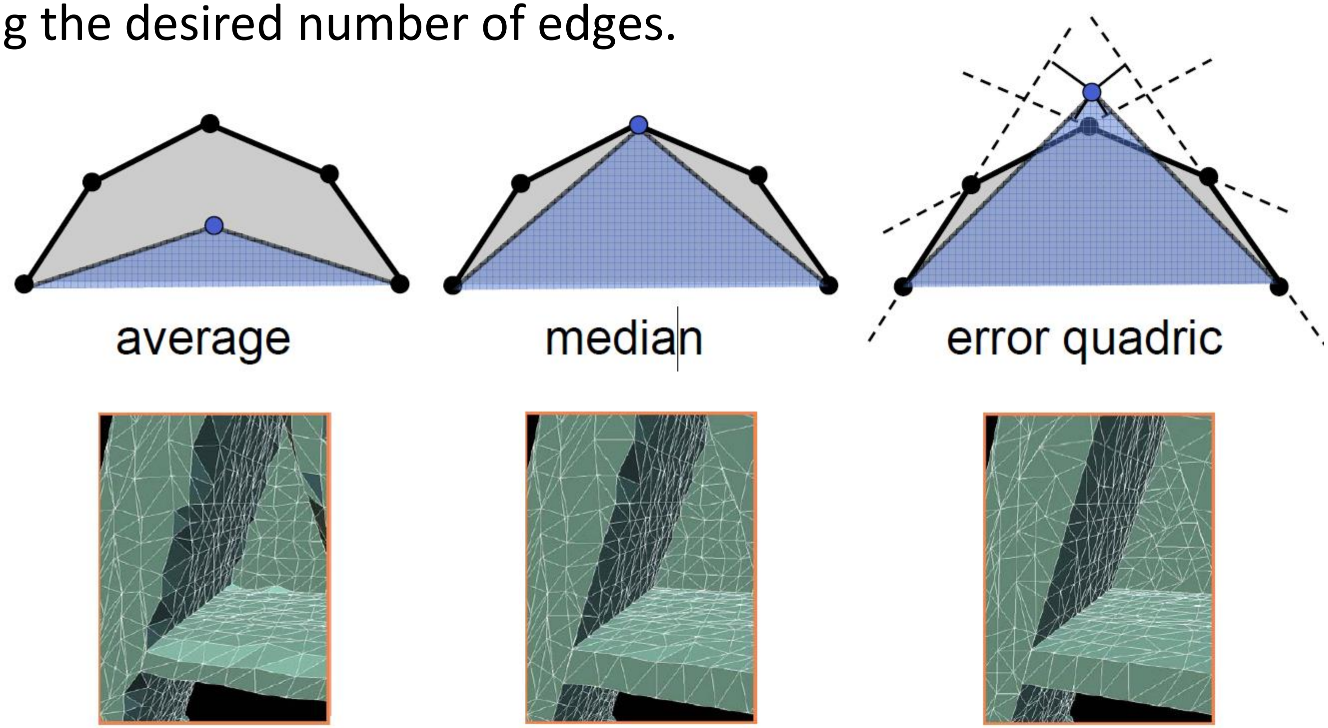


Mesh Simplification

- Surface mesh simplification is the process of reducing the number of faces used in a surface mesh while keeping the overall shape, volume and boundaries preserved as much as possible. It is the opposite of subdivision.

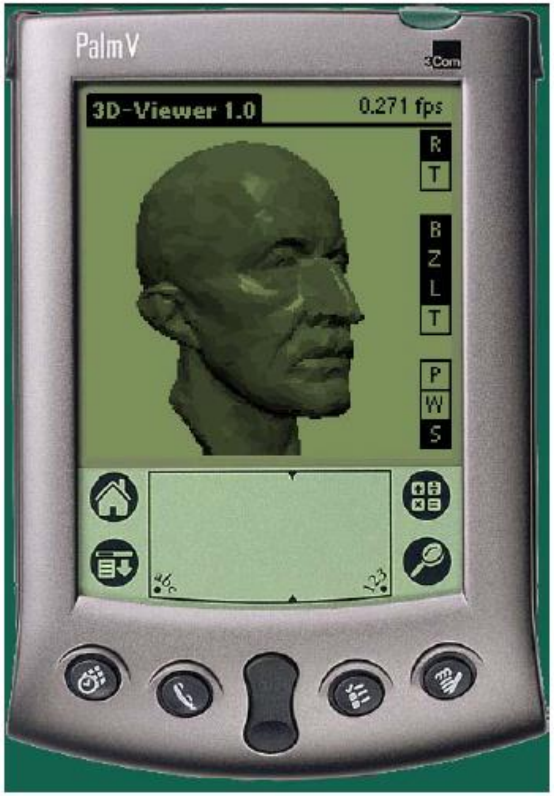
Mesh Simplification

- Edges are collapsed according to a priority given by a user-supplied cost function, and the coordinates of the replacing vertex are determined by another user-supplied placement function.
- The algorithm terminates when a user-supplied stop predicate is met, such as reaching the desired number of edges.



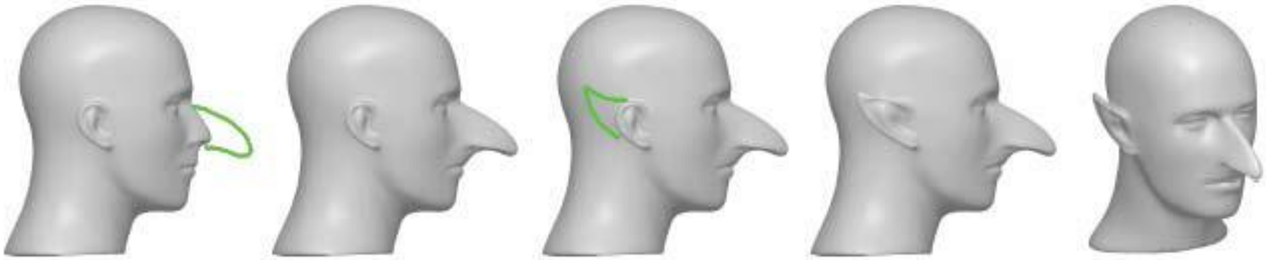
Mesh Simplification

- Adaptation to hardware capabilities



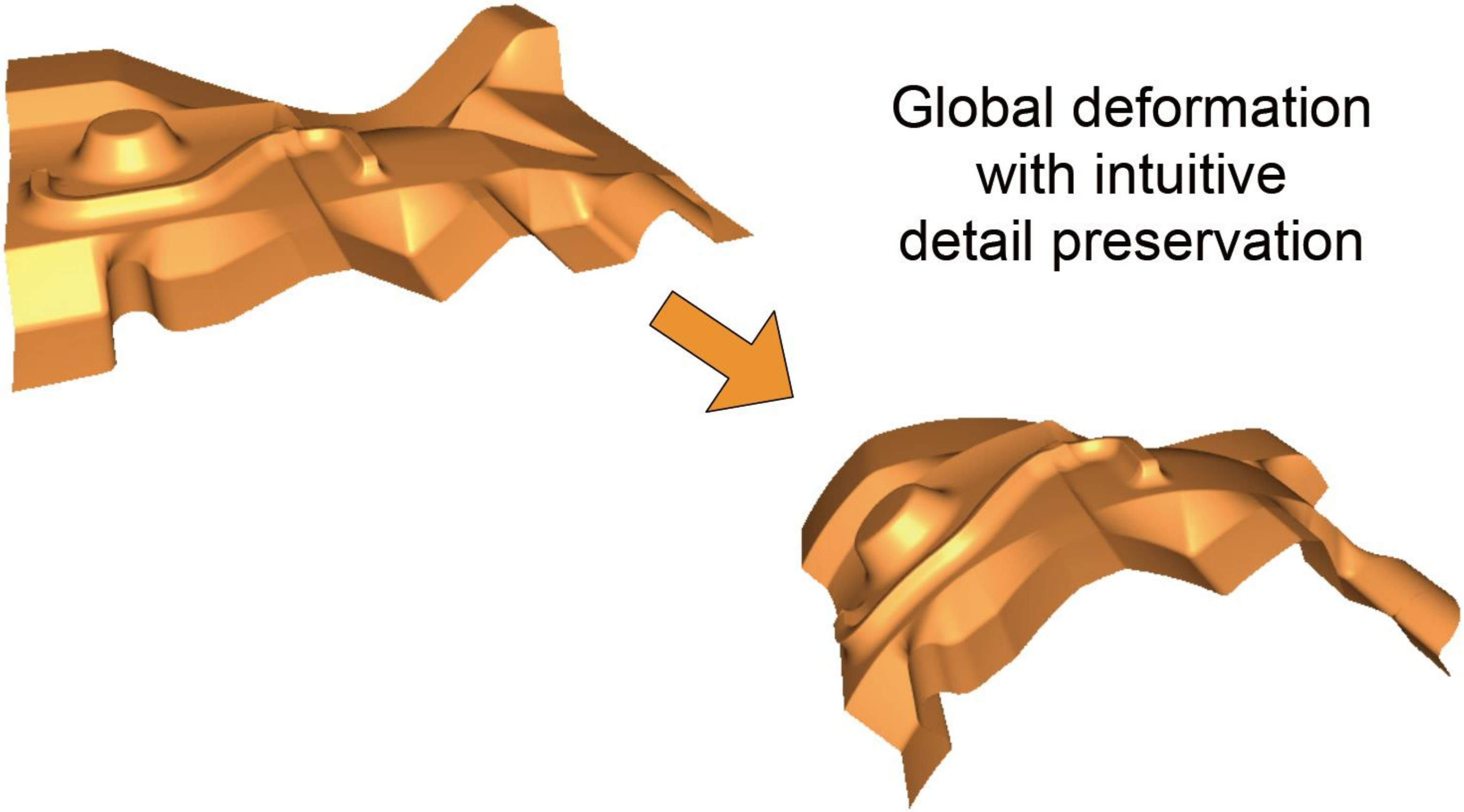
Shapes and Deformations

- Why deformations?
  - Sculpting, customization
  - Character posing, animation
- Criteria?
  - Intuitive behavior and interface
  - Interactivity

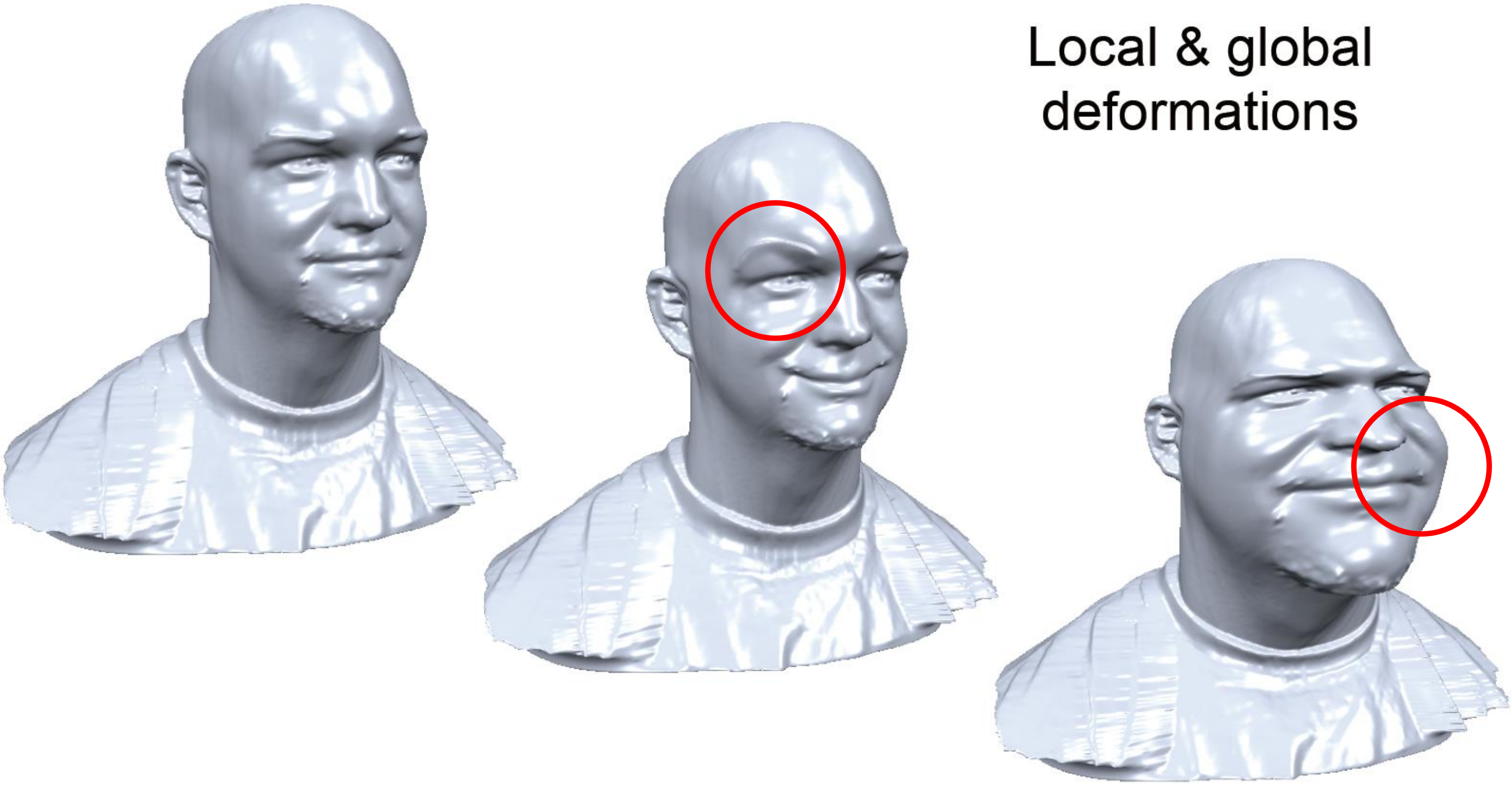


Linear Surface-Based Deformation

- Mesh Deformation



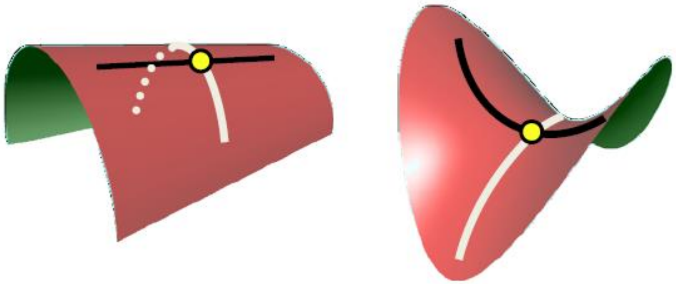
Mesh Deformation

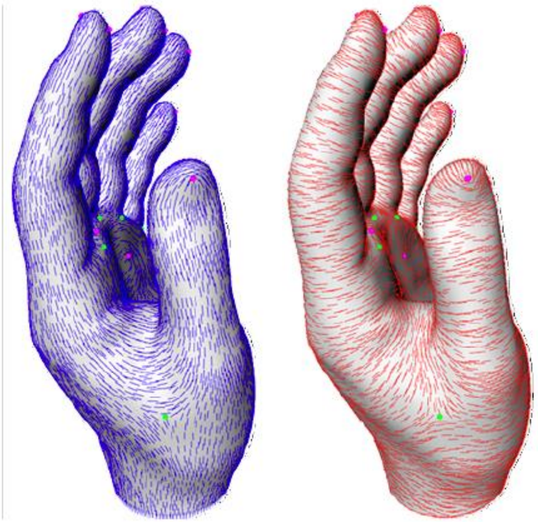


Local & global deformations

Differential Geometry

- Tool to analyze shape
- Key notions:
  - Tangents and normals
  - Curvatures
  - Laplace-Beltrami operator





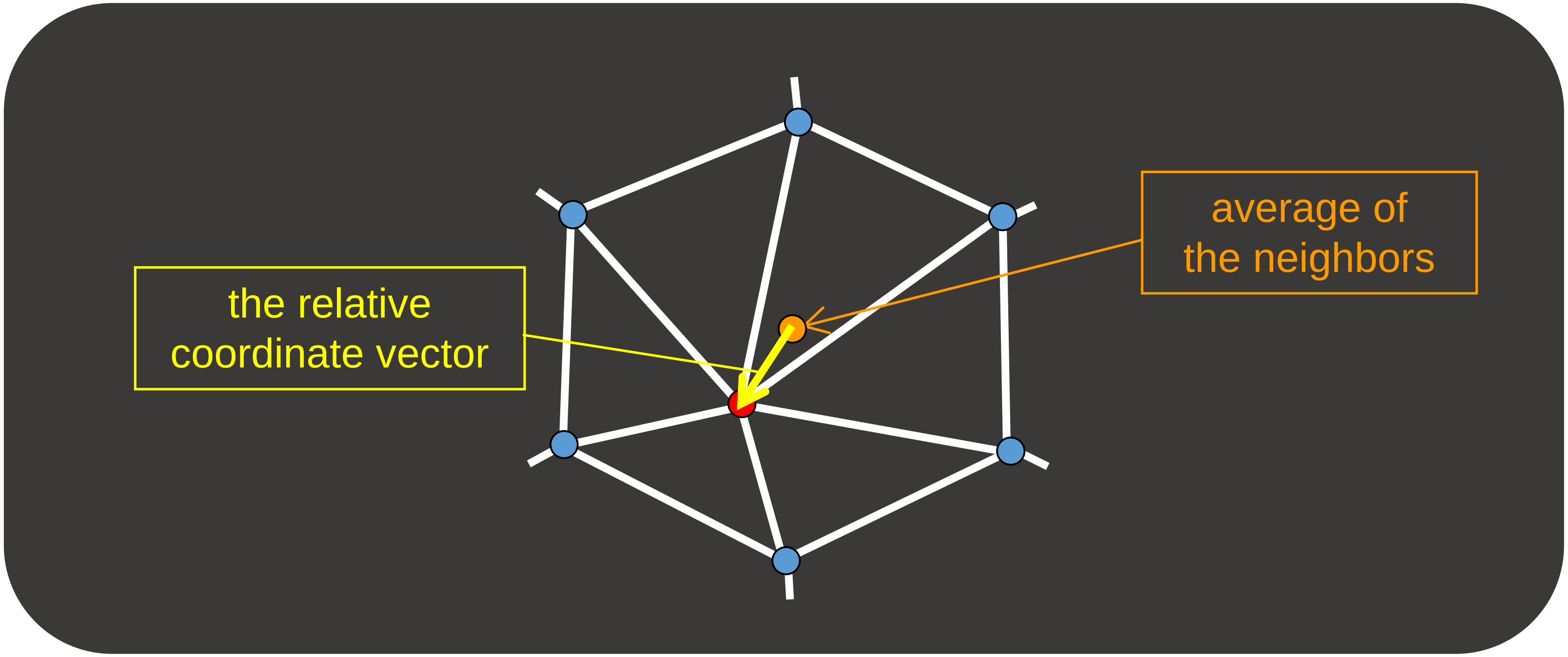
Differential Coordinates

- Manipulate differential coordinates instead of spatial coordinates
  - Gradients, Laplacians, local frames
  - Intuition: Close connection to surface normal
- Find mesh with desired differential coords
  - Cannot be solved exactly
  - Formulate as energy minimization

Differential coordinates

- Differential coordinates are defined for triangular mesh vertices

$$\boldsymbol{\delta}_{\mathbf{i}} = L(\mathbf{v}_{\mathbf{i}}) = \mathbf{v}_{\mathbf{i}} - \frac{1}{d_i} \sum_{j \in N(i)} \mathbf{v}_{\mathbf{j}}$$



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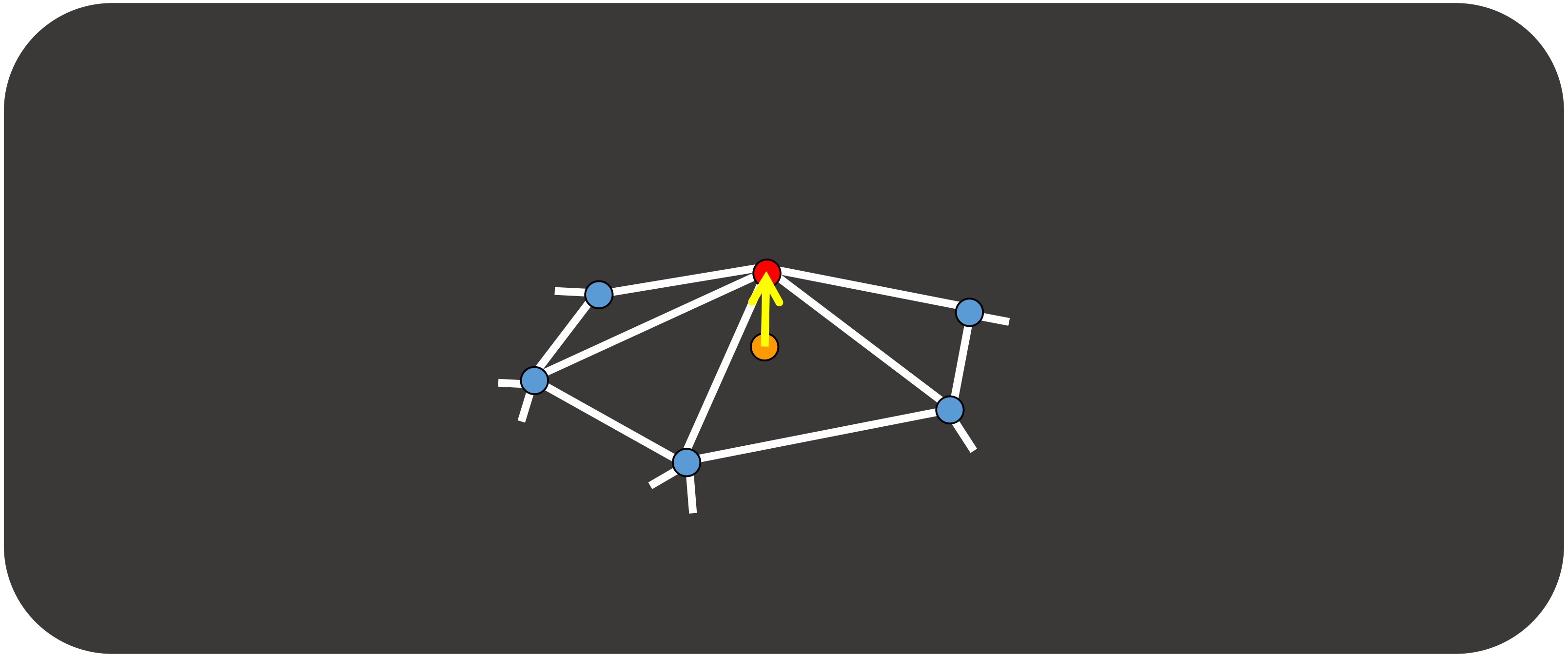
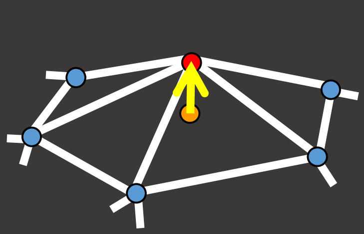
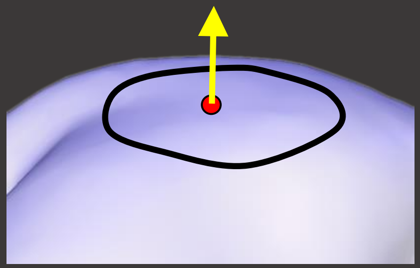


Why differential coordinates?

- They represent the local detail / local shape description
  - The direction approximates the normal
  - The size approximates the mean curvature





$$\boldsymbol{\delta}_{\mathrm{i}} = \frac{1}{d_i} \sum_{\mathbf{v} \in N(i)} \left(\mathbf{v}_{\mathrm{i}} - \mathbf{v} \right)$$

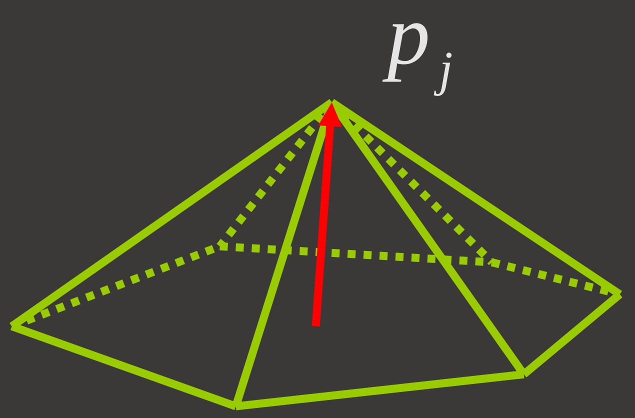
$$\frac{1}{len(\gamma)} \int_{\mathbf{v} \in \gamma} \left(\mathbf{v}_{\mathrm{i}} - \mathbf{v} \right) ds$$

$$\lim_{len(\gamma) \to 0} \frac{1}{len(\gamma)} \int_{\mathbf{v} \in \gamma} \left(\mathbf{v}_{\mathrm{i}} - \mathbf{v} \right) ds = H(\mathbf{v}_{\mathrm{i}}) \mathbf{n}_{\mathrm{i}}$$

Laplacian reconstruction

- Denote by $G = (V, E, P)$ a triangular mesh with geometry P, embedded in R^3 .
- For each vertex $p_j \in P$ we define the Laplacian vector:

$$L_j(P) = p_j - \frac{1}{d_j} \sum_{i:(i,j) \in E} p_i$$



- The Laplacians represents the details locally.

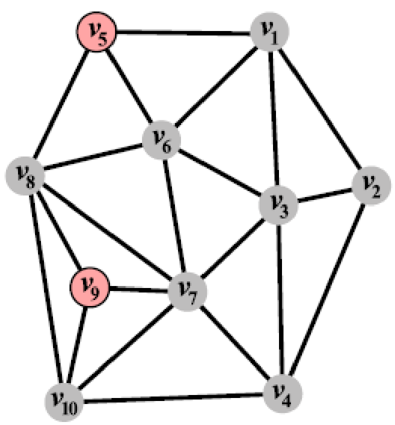
Laplacian reconstruction

- The operator L is linear and thus can be represented by the following matrix:

$$M_{ij} = \begin{cases} 1 & i = j \\ -\dfrac{1}{d_i} & j \in \{ j : (j,i) \in E \} \\ 0 & otherwise \end{cases}$$

Laplacian reconstruction

- A small example of a triangular mesh and its associated Laplacian matrix



The mesh

$$\begin{bmatrix} 4 & -1 & -1 & & -1 & -1 & & & & \\ -1 & 3 & -1 & -1 & & & & & & \\ -1 & -1 & 5 & -1 & & -1 & -1 & & & \\ & -1 & -1 & 4 & & & -1 & & & -1 \\ -1 & & & & 3 & -1 & & -1 & & \\ -1 & & -1 & & & 4 & -1 & -1 & & \\ & & -1 & -1 & & -1 & 6 & -1 & -1 & -1 \\ & & & & -1 & -1 & -1 & 6 & -1 & -1 \\ & & & & & & -1 & -1 & 3 & -1 \\ & & & -1 & & & -1 & -1 & -1 & 4 \end{bmatrix}$$

The symmetric Laplacian L_s

$$\begin{bmatrix} 4 & -1 & -1 & & & -1 & & & & \\ -1 & 3 & -1 & -1 & & & & & & \\ -1 & -1 & 5 & -1 & & -1 & -1 & & & \\ & -1 & -1 & 4 & & & -1 & & & -1 \\ & & & & & & & & & \\ -1 & & -1 & & & 4 & -1 & -1 & & \\ & & -1 & -1 & & -1 & 6 & -1 & & -1 \\ & & & & & -1 & -1 & 6 & & -1 \\ & & & & & & & & & \\ & & & -1 & & & -1 & -1 & & 4 \end{bmatrix}$$

Invertible Laplacian

$$\begin{bmatrix} 4 & -1 & -1 & & -1 & -1 & & & & \\ -1 & 3 & -1 & -1 & & & & & & \\ -1 & -1 & 5 & -1 & & -1 & -1 & & & \\ & -1 & -1 & 4 & & & -1 & & & -1 \\ -1 & & & & 3 & -1 & & -1 & & \\ -1 & & -1 & & & 4 & -1 & -1 & & \\ & & -1 & -1 & & -1 & 6 & -1 & -1 & -1 \\ & & & & -1 & -1 & -1 & 6 & -1 & -1 \\ & & & & & & -1 & -1 & 3 & -1 \\ & & & -1 & & & -1 & -1 & -1 & 4 \\ & & & & 1 & & & & & \\ & & & & & & & & 1 & \end{bmatrix}$$

2-anchor $\tilde{L}$

Laplacian reconstruction

- Thus for reconstructing the mesh from the Laplacian representation:

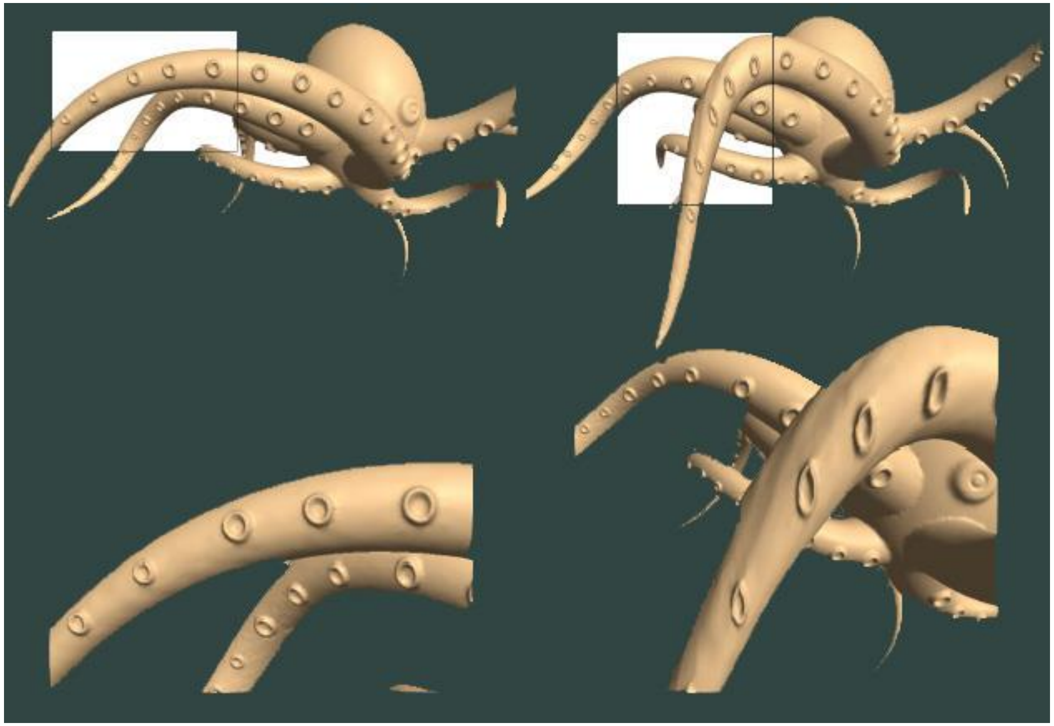
add constraints to get full rank system and therefore unique solution, i.e. unique minimizer to the functional

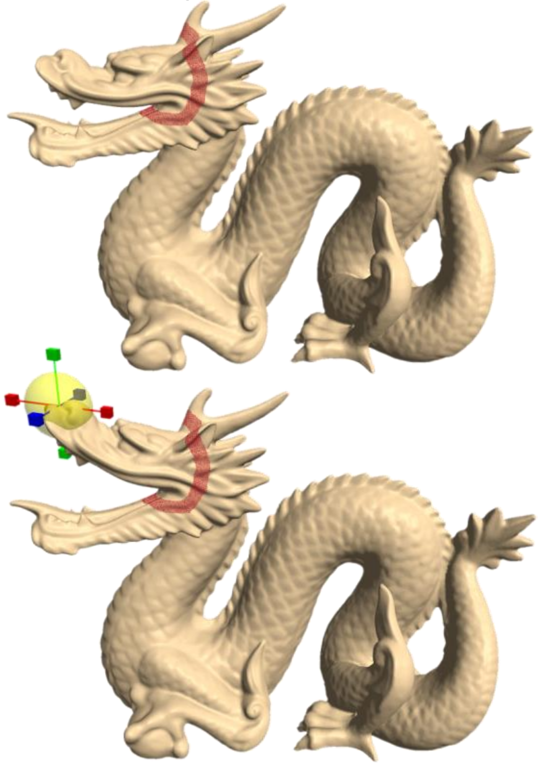
$$\left\| M \cdot P^{(x)} - \delta^{(x)} \right\|^2 + \sum_{i \in I} w_i \left(p_i^{(x)} - c_i^{(x)} \right)^2$$

where I is the index set of constrained vertices , $w_i > 0$ are weights and c_i are the spatial constraints.

Laplacian reconstruction

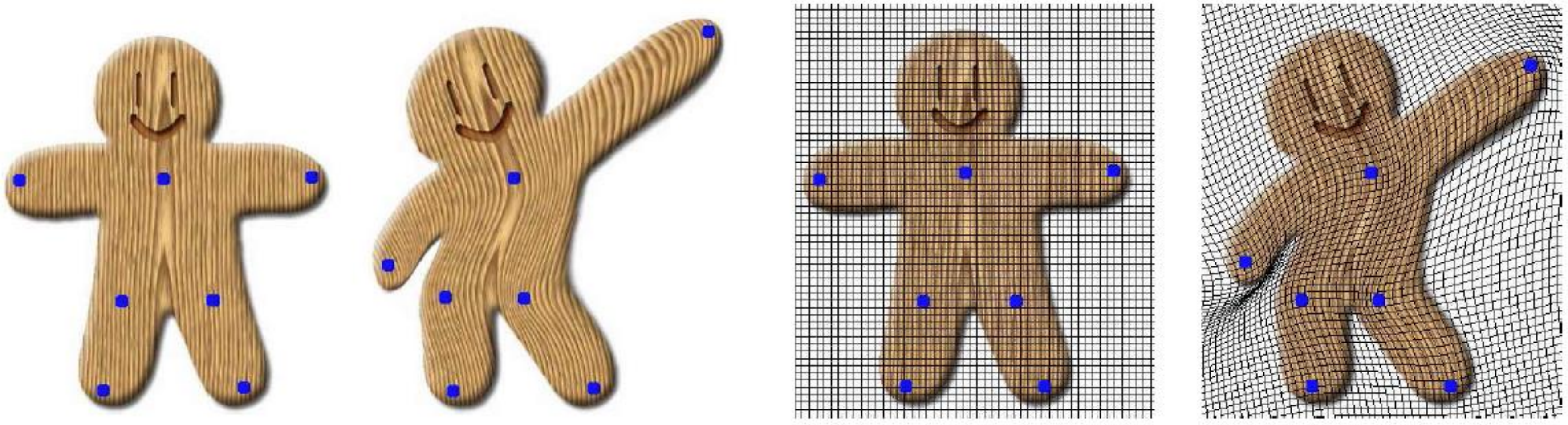
- Laplacian reconstruction gives smooth transformation, interactive time and ease of user interface -using few spatial constraints
- but doesn’t preserve details orientation and shape





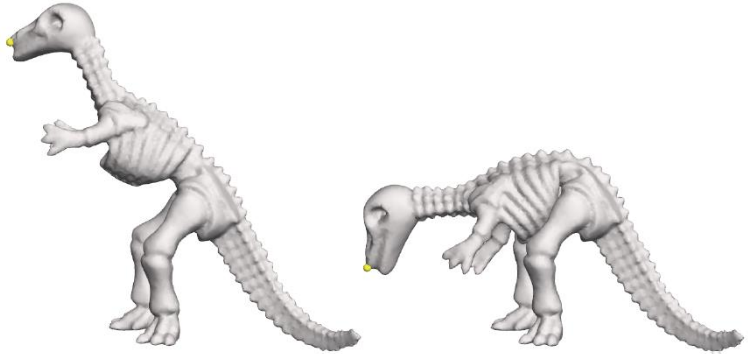
As-Rigid-As-Possible Deformation

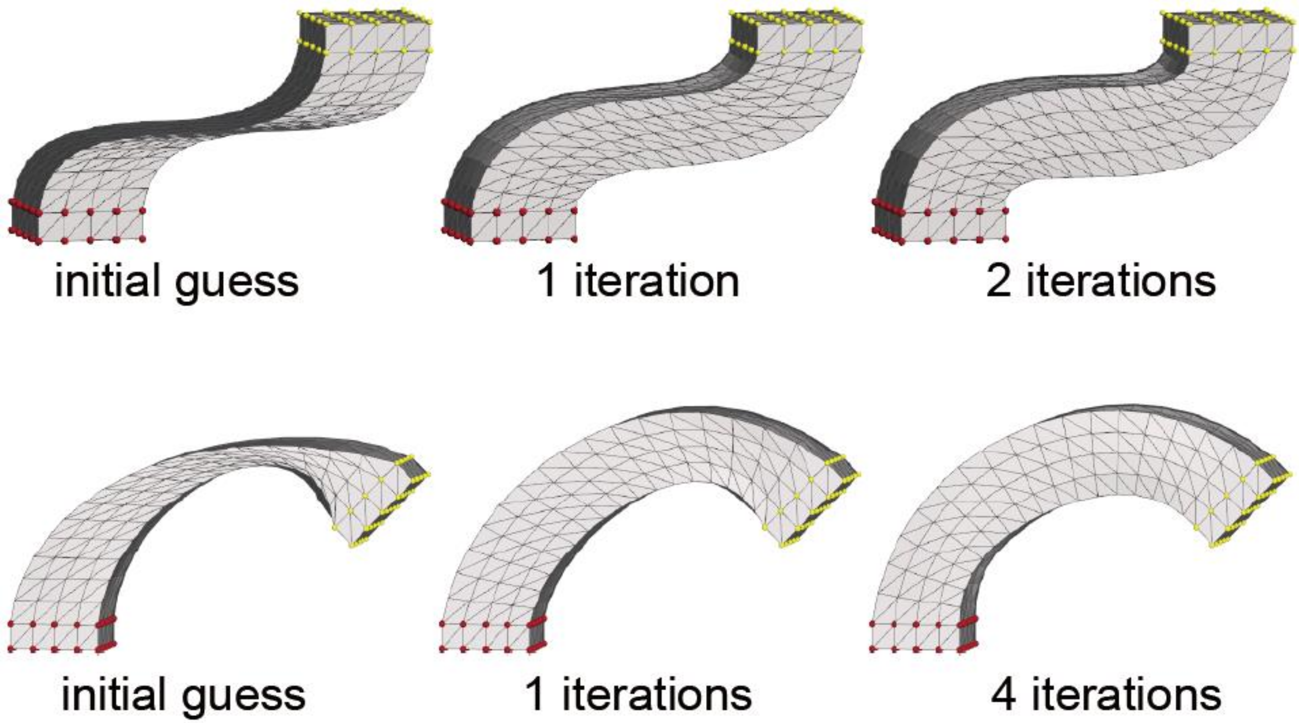
- Points or segments as control objects
- First developed in 2D and later extended to 3D by Zhu and Gortler (2007)



As-Rigid-As-Possible Deformation

- Smooth large scale deformation
- Local as-rigid-as-possible behavior
  - Preserves small-scale details





Space Deformation

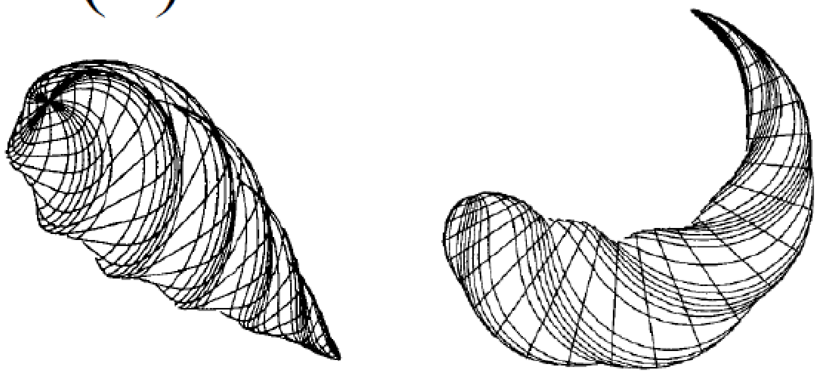
- Displacement function defined on the ambient space

$$\mathbf{d} : \mathrm{R}^3 \rightarrow \mathrm{R}^3$$

- Evaluate the function on the points of the shape embedded in the space

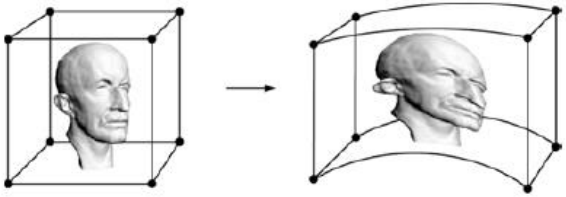
$$\mathbf{x}' = \mathbf{x} + \mathbf{d}(\mathbf{x})$$

Twist warp
Global and local deformation of solids
[A. Barr, SIGGRAPH 84]

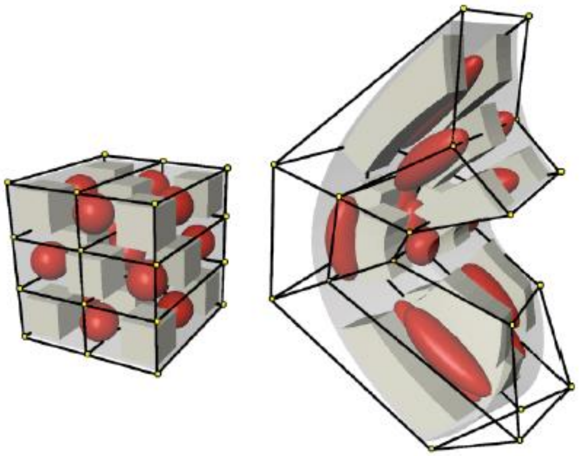


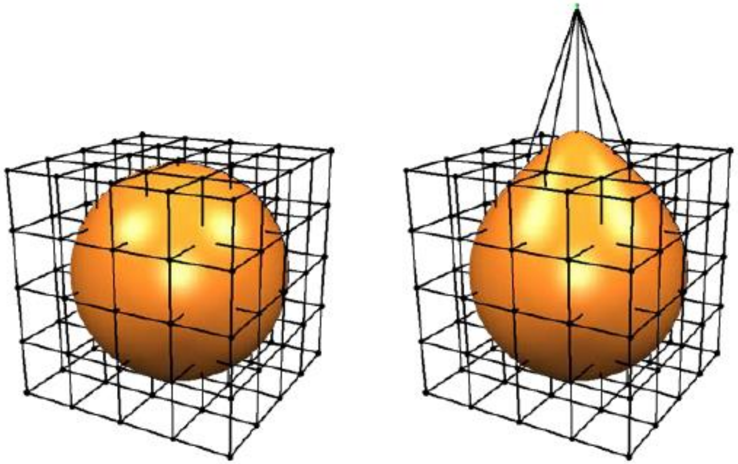
Space Deformation

- Control object = lattice
- Basis functions $B_i(\mathbf{x})$ are trivariate tensor-product splines:



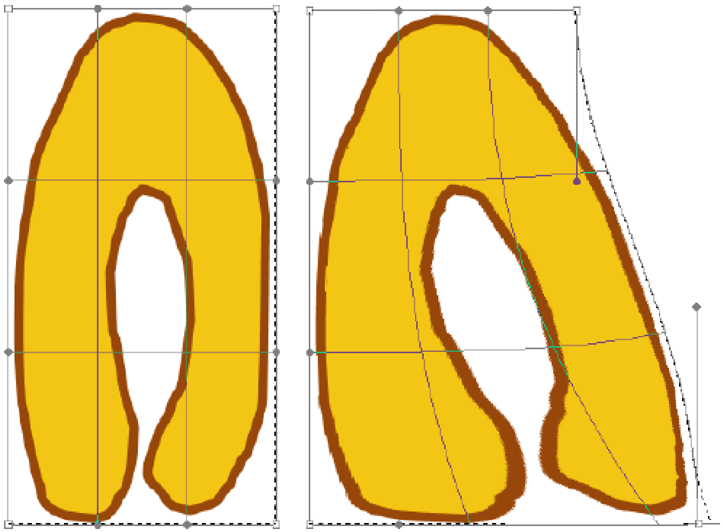
$$\mathbf{d}(x,y,z)=\sum_{i=0}^{l}\sum_{j=0}^{m}\sum_{k=0}^{n}\mathbf{d}_{ijk}\,N_i(x)N_j(y)N_i(z)$$





Lattice as Control Object

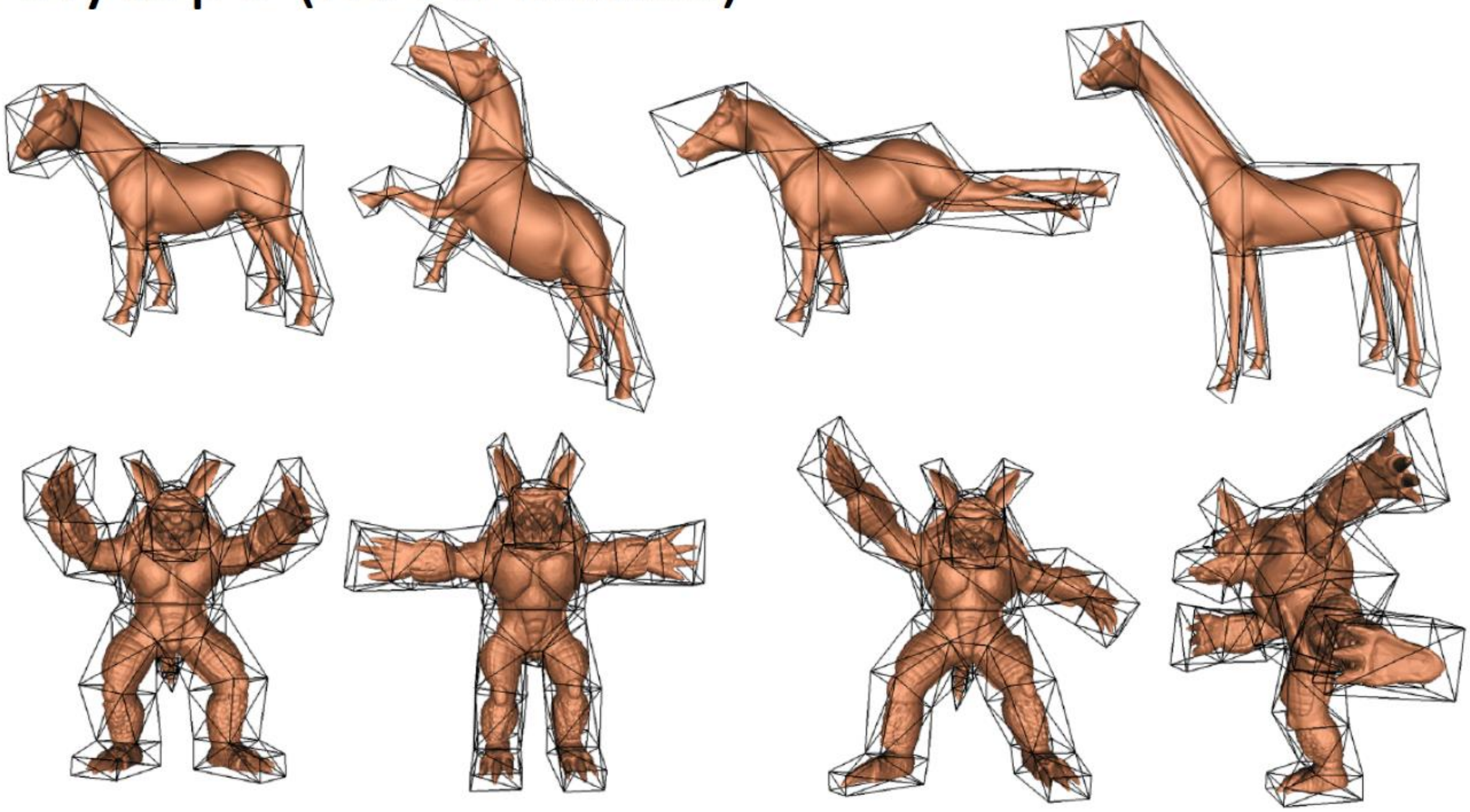
- Difficult to manipulate
- The control object is not related to the shape of the edited object
- Part of the shape in close Euclidean distance always deform similarly, even if geodesically far



Cage-based Deformations

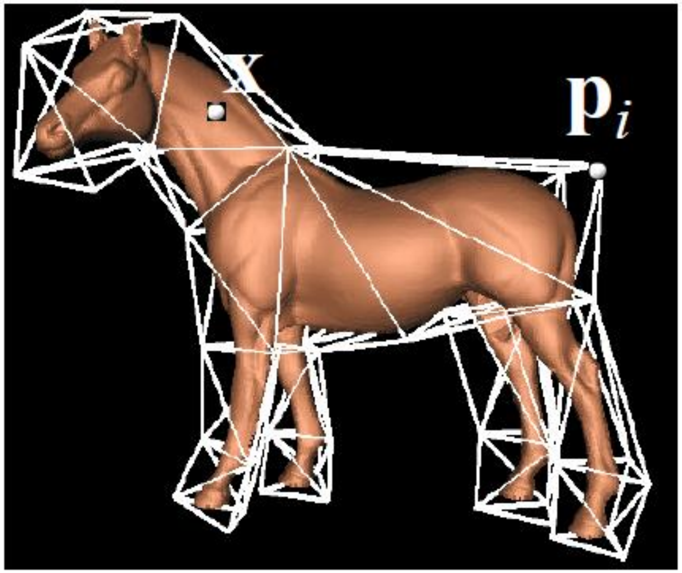
- Cage = crude version of the input shape
- Polytope (not a lattice)

[Ju et al. 2005]



Cage-based Deformations

- Cage = crude version of the input shape
- Polytope (not a lattice)
- Each point $\mathbf{x}$ in space is represented w.r.t. to the cage elements using coordinate functions



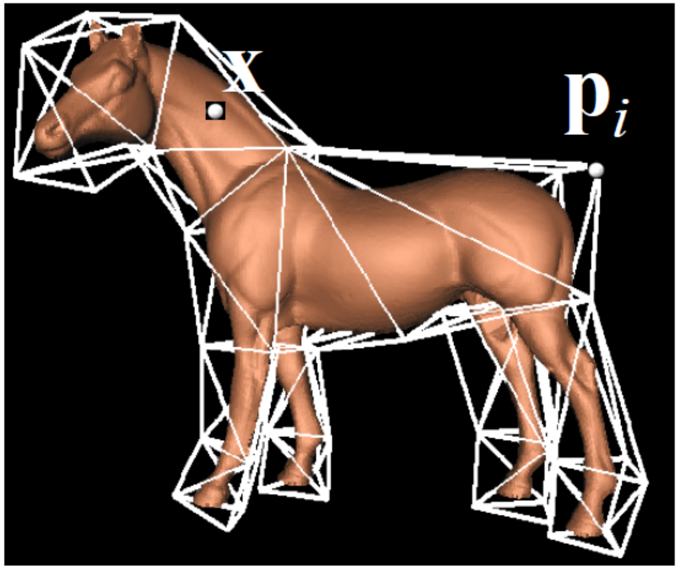
$$\mathbf{x} = \sum_{i=1}^{k} w_i(\mathbf{x})\mathbf{p}_i$$

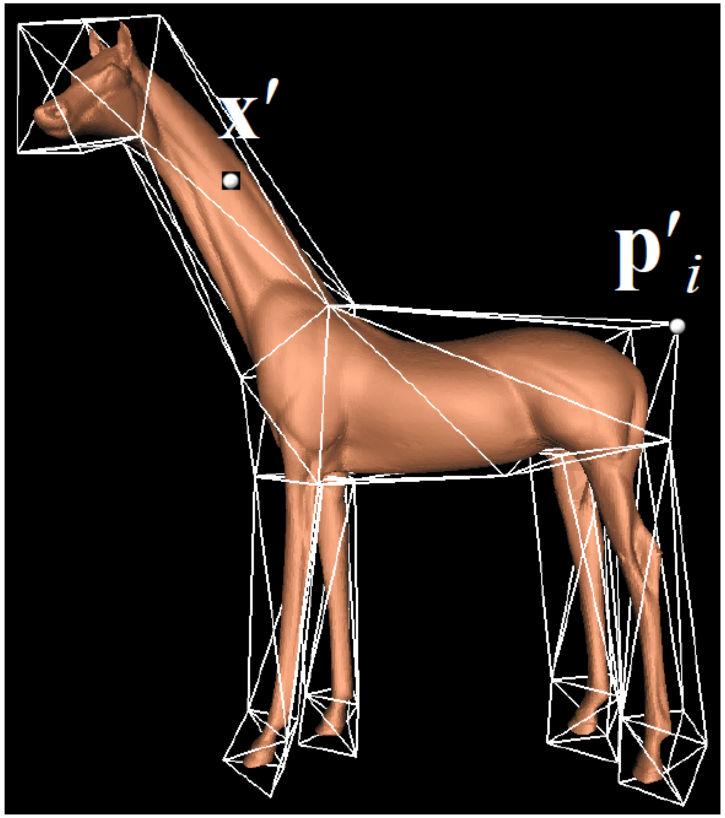
[Ju et al. 2005]

Cage-based Deformations

- Cage = crude version of the input shape
- Polytope (not a lattice)

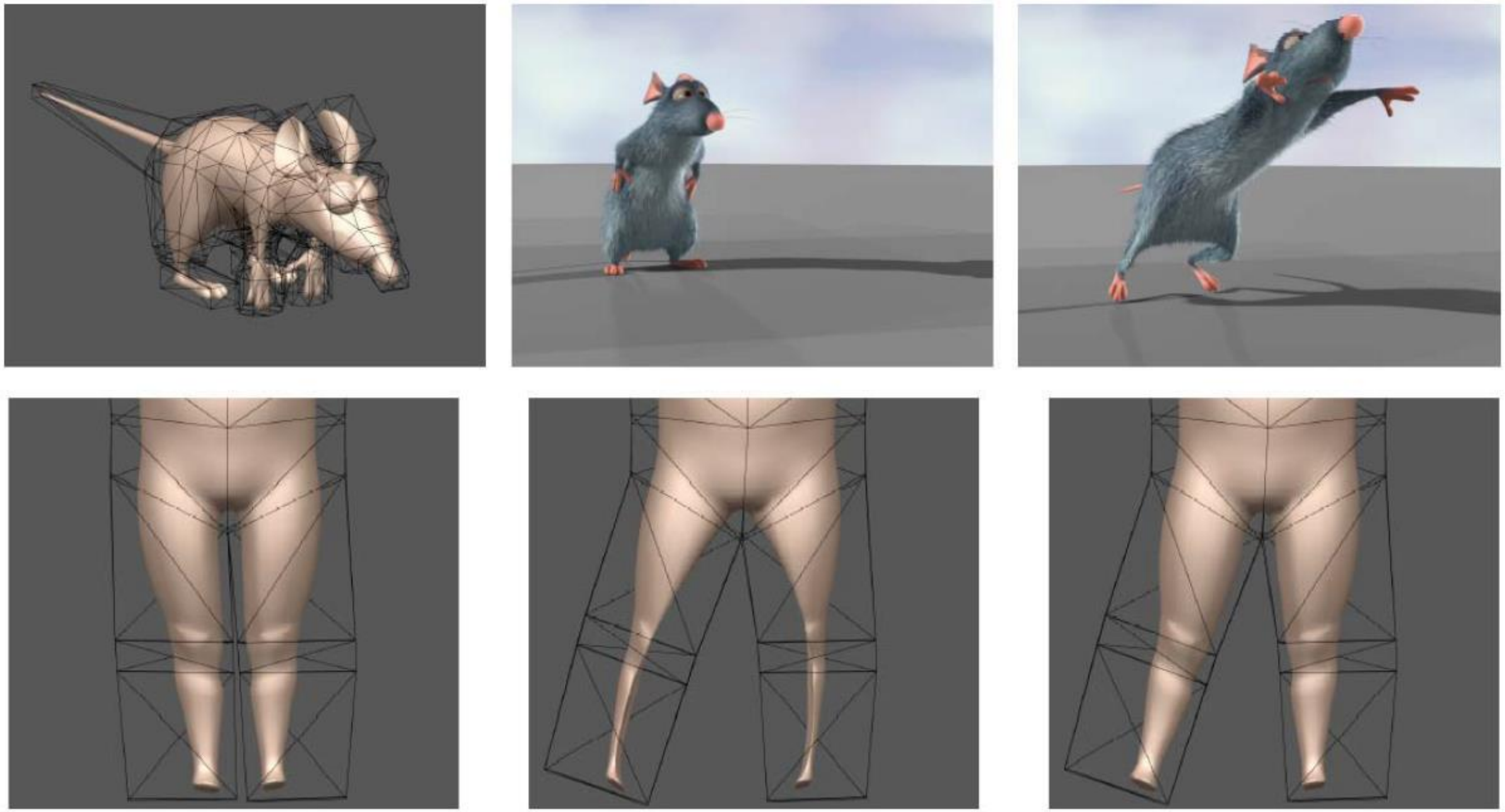
$$\mathbf{x}' = \sum_{i=1}^{k} w_i(\mathbf{x})\mathbf{p}'_i$$





Coordinate Functions

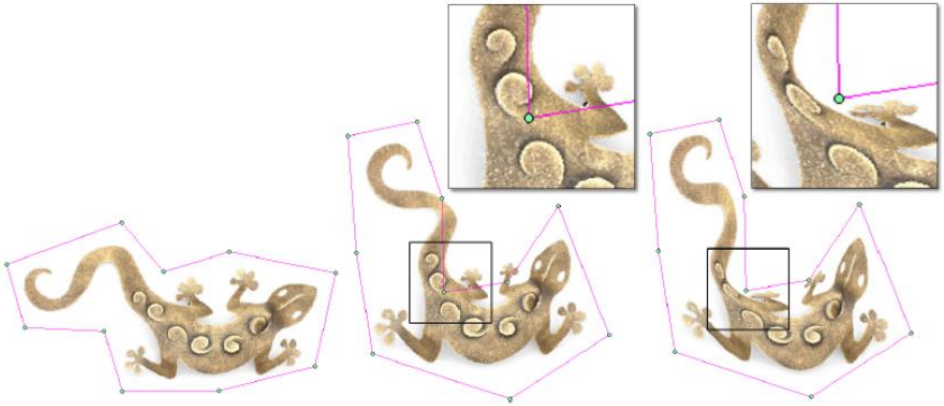
- Harmonic coordinates (Joshi et al. 2007)



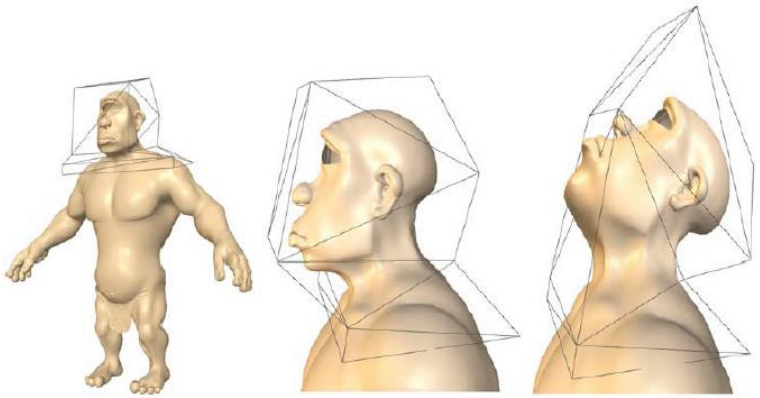
MVC HC

Green coordinates

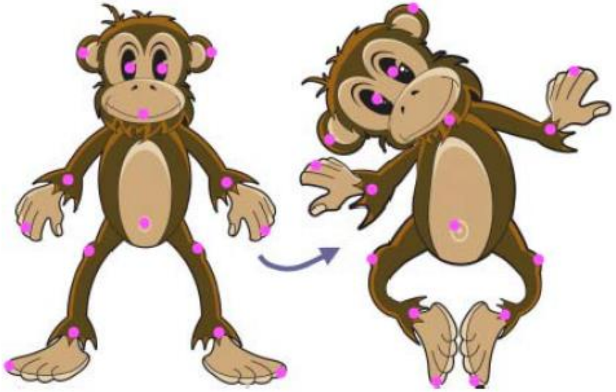
- Closed-form solution
- Conformal in 2D, quasi-conformal in 3D

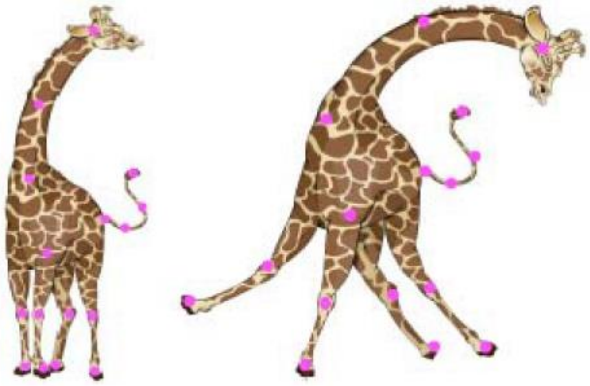


GC MVC



GC





Polygon Mesh Processing

- http://www.pmp-book.org/
- "Geometric Modeling Based on Polygonal Meshes"
- https://hal.inria.fr/inria-00186820/document